MTH 264, Quiz #7, Fall 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Consider the sequence $a_n = \dfrac{5n}{\sqrt{n^2+4}}$.

   a. Determine whether the sequence is bounded.

   ✓ BOUNDED ABOVE AND BELOW below by 0 $n \geq 0$, above by 5

   BOUNDED ABOVE ONLY

   BOUNDED BELOW ONLY

   NOT BOUNDED ABOVE OR BELOW

   b. Is the sequence monotonic? yes

   c. Does the sequence converge or diverge? converges

   d. If the sequence converges, state its limit.

   $$\lim_{n\to\infty} \frac{5n}{\sqrt{n^2+4}} = 5$$

2. Come up for a formula for the nth term of the sequence $\frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \frac{63}{32}, \ldots$. Test it to be sure it works.

   $$a_n = \frac{2^{n+1}-1}{2^n}$$ Start at $n=1$

   $3 = 4-1$
   $7 = 8-1$
   $15 = 16-1$
   $31 = 32-1$
   $63 = 64-1$

3. Find the first 5 terms of the partial sum and find an expression for S_n for the series $\sum_{k=1}^{\infty} \frac{2}{(2k+3)(2k+1)}$.

$$\frac{2}{5(3)} + \frac{2}{7(5)} + \frac{2}{9(7)} + \frac{2}{11(9)} + \frac{2}{13(11)}$$

$$= \sum_{k=1}^{\infty} \left[\frac{1}{2k+1} - \frac{1}{2k+3} \right]$$

$$\frac{A}{2k+3} + \frac{B}{2k+1} = A(2k+1) + B(2k+3)$$

$$2Ak + A + 2Bk + 3B = 2$$

$$2A + 2B = 0 \to A + B = 0$$

$$A + 3B = 2$$

$$-A - B = 0$$

$$2B = 2$$

$$B = 1 \quad A = -1$$

$S_n =$

$$\frac{1}{3} - \frac{1}{2n+3}$$

$$\frac{1}{3} - \frac{1}{5} = S_1$$

$$\frac{1}{3} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \frac{1}{7} = S_2$$

$$\frac{1}{3} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} - \frac{1}{9} = S_3$$