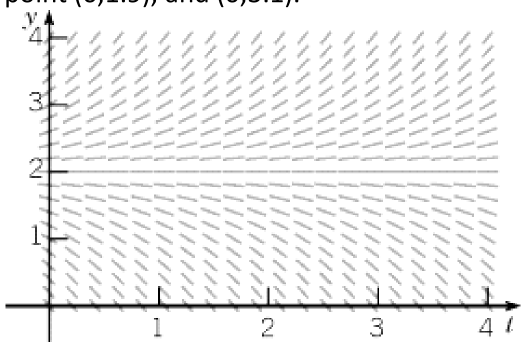
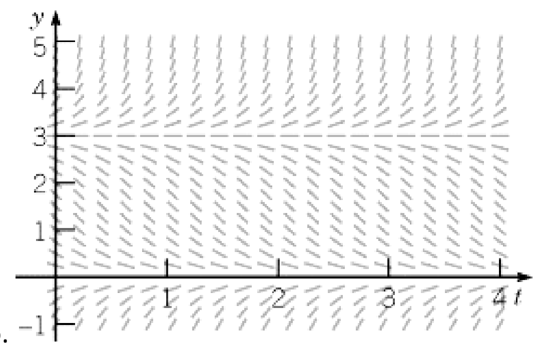
MTH 267, Homework #4, Fall 2022   Name ____________________________________

Instructions: Work problems on a separate sheet of paper and attach work to this page. You should show all work to receive full credit for problems. Checking your work with computer algebra systems is fine, but that doesn't count as "work" since you won't be able to use CAS programs on exams or quizzes. Graphs and longer answers that won't fit here, indicate which page of the work the answer can be found on and be sure to clearly indicate it on the attached pages.

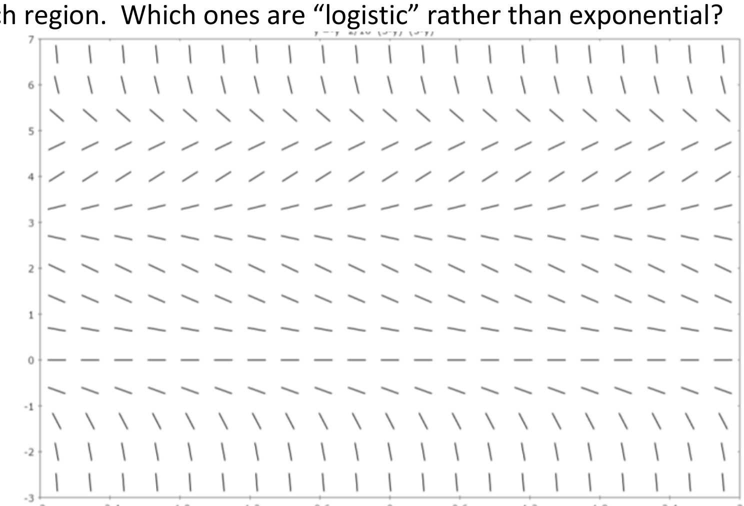
1. Graph the direction field for each autonomous equation by hand and comment on the stability of each equilibrium.
   a. $y' = 1 + 2y$ b. $y' = -y(5 - y)$ c. $y' = y(y-2)^2$

2. For the direction fields below, state the differential equation that produces the graph (assume the equilibria are integer values). Plot the path a particle would take in the field if it started at the point (0,1.9), and (0,3.1).

a. 

b. 

3. A direction field for the differential equation $y' = -\frac{1}{10}y^2(3-y)(5-y)$ is shown below. For each stationary point, describe it as a carrying capacity, a threshold, or neither. Graph a sample trajectory in each region. Which ones are "logistic" rather than exponential?



4. For each of the differential equations below, estimate the solutions using the indicated step size, for 5 steps. First, estimate the solution with the Improved Euler's method, and then use the improved Euler's method. After doing the first 5 steps by hand, use a computer to plot 25 more steps and graph the estimated solution.

a. $y' = \sqrt{t+y}, y(0) = 3, h = 0.025$
b. $y' = \frac{4-ty}{1+y^2}, y(0) = -2, h = 0.0125$
c. $y' = 2y - 3t, y(0) = 1, h = 0.05$

5. For each of the differential equations below, estimate the solutions using the indicated step size, for 5 steps. Estimate the solution using the Runge-Kutta method. After doing the first 5 steps by hand, use a computer to plot 25 more steps and graph the estimated solution.
   d. $y' = \sqrt{t+y}, y(0) = 3, h = 0.025$
   e. $y' = \frac{4-ty}{1+y^2}, y(0) = -2, h = 0.0125$
   f. $y' = 2y - 3t, y(0) = 1, h = 0.05$

6. Use Runge-Kutta to estimate the solutions to the system of differential equations. Estimate the solution to $y(1)$ using $h = 0.25$.
   a. $x' = x + y + t, y' = 4x - 2y, x(0) = 1, y(0) = 0$
   b. $x' = x - y + xy, y' = 3x - 2y - xy, x(0) = 0, y(0) = 1$

7. Prove that the following differential equations are exact, and then solve the equation. If initial conditions are provided, solve for the constant.
   a. $(3x^2 - 2xy + 2)dx + (6y^2 - x^2 + 3)dy = 0$
   b. $(9x^2 + y - 1)dx - (4y - x)dy = 0, y(1) = 0$
   c. $(ye^{xy}\cos 2x - 2e^{xy}\sin 2x + 2x)dx + (xe^{xy}\cos 2x - 3)dy = 0$

8. The following equations are not exact, but can be made exact with an appropriate integrating factor. Find that integrating factor, and then solve the resulting exact equation.
   a. $ydx + (2x - ye^y)dy = 0, \mu(x,y) = y$ b. $dx + \left(\frac{x}{y} - \sin y\right)dy = 0$