

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Rewrite $u'' + \frac{1}{2}u' + 2u = 0$ as a system of first order equations.

2. Rewrite the system $\begin{cases} x_1' = 3x_1 - 2x_2 \\ x_2' = 2x_1 - 2x_2 \end{cases}$ as a single second order equation.

3. Find the eigenvalues and eigenvectors of

a. $A = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$

b. $B = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$

4. Solve $\vec{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \vec{x}$, $\vec{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ [Hint: see #3a]

5. Draw a direction field for the system $\vec{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \vec{x}$ and plot several trajectories of the system.