MTH 267, Quiz #2, Fall 2022 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use Euler's method to find $y(1)$ for the differential equation $\frac{dy}{dt} = y(y-2t)$, $y(0) = -2$. Use $\Delta t = 0.05$. Verify two steps of your calculation by hand, and then complete the remaining steps with technology (such as Excel). Plot the resulting curve.

See Excel

$n=0 \quad m_0 = -2(-2-0) = 4$

$y_1 = 4(0.05) + -2 = -1.8$

$n=1 \quad m_1 = -1.8(-1.8 - 2(0.05)) = 3.42$

$y_2 = 3.42(0.05) - 1.8 = -1.629$

2. Solve the differential equation $\frac{dy}{dt} = 4 + y$ for the analytic solution. Solve for the missing constant if the initial condition is y(0)=1. (Use separation of variables.)

$\int \frac{dy}{4+y} = \int dt$

$\ln|y+4| = t + C$

$y + 4 = e^{t+C} \Rightarrow y+4 = Ae^t$

$y = Ae^t - 4$

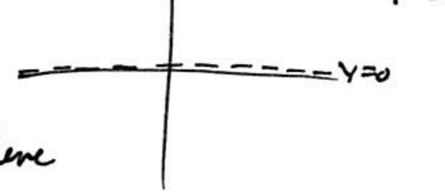
$1 = Ae^0 - 4$

$1 = A - 4$

$5 = A$

$\boxed{y = 5A^t - 4}$

3. For the ODE $\frac{dy}{dt} = \frac{1+t^2}{3y-y^2}$, determine where a solution exists. Sketch the region in the plane. (Be sure to show explicitly that you check **both** conditions.)



Solution exists where
$\{(x,y) \mid y \neq 0, 3\}$

$3y - y^2 = 0$

$y(3-y) = 0 \quad y=0, y=3$

undefined

$f = (1+t^2)(3y-y^2)^{-1}$

$f' = (1+t^2)(-1)(3y-y^2)^{-2}(3-2y)$

$= \frac{(1+t^2)(2y-3)}{(3y-y^2)^2}$ undef. at $y=0$, $y=3$