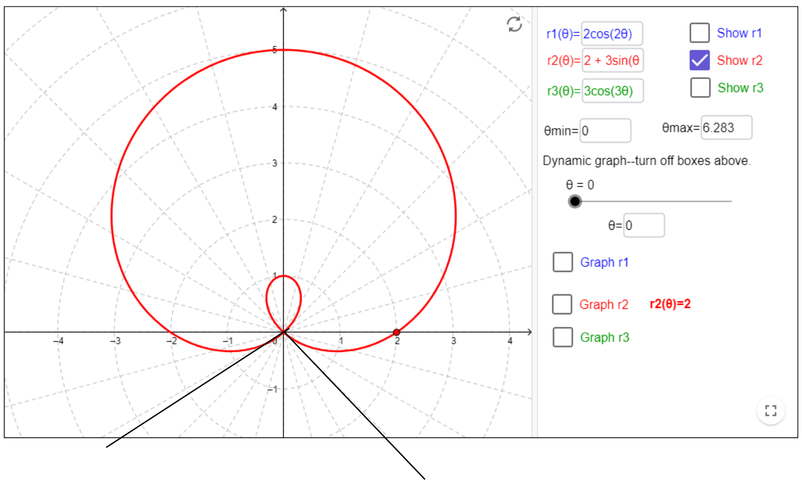
12/5/2023

Review for Final

Quiz #15 Problem #2

Find the area of the inner loop of $r = 2 + 3\sin\theta$



Find the beginning and end of the loop:

$$2 + 3\sin\theta = 0$$
$$\theta = \sin^{-1}\left(-\frac{2}{3}\right)$$

Reference angle: $\sin^{-1}\left(\frac{2}{3}\right)$

Third quadrant angle is $\pi + ref.angle = \pi + \sin^{-1}\left(\frac{2}{3}\right) = \theta_1$

Fourth quadrant angle $2\pi - ref.angle = 2\pi - \sin^{-1}\left(\frac{2}{3}\right) = \theta_2$

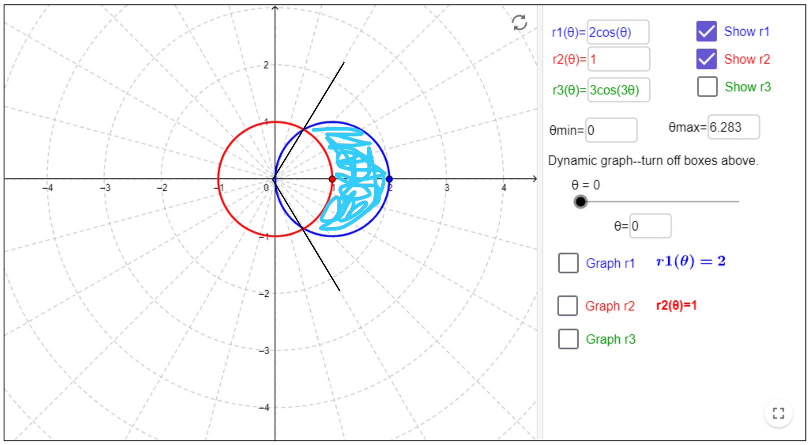
Can convert to radians.

$$A = \frac{1}{2}\int_{\theta_2}^{\theta_1} [r(\theta)]^2 d\theta = \frac{1}{2}\int_{\theta_2}^{\theta_1} [2 + 3\sin\theta]^2 d\theta = \frac{1}{2}\int_{\theta_2}^{\theta_1} 4 + 12\sin\theta + 9\sin^2\theta \, d\theta$$

At least complete up to the integration.

Quiz #15 Problem #3

Find the area inside $r = 2\cos\theta$ and outside $r = 1$.

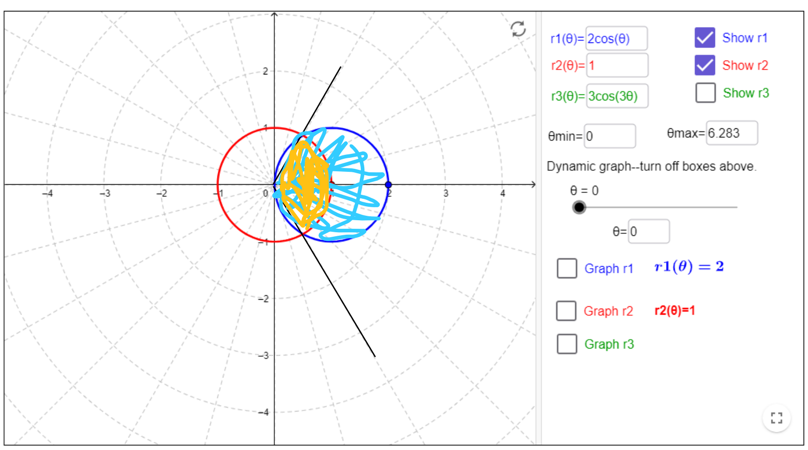


Intersection:

$$2\cos\theta = 1$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}, -\frac{\pi}{3}$$

Use symmetry: instead of using $-\frac{\pi}{3}$ to $\frac{\pi}{3}$ integrate from 0 to $\frac{\pi}{3}$ and double it.

$$A = \frac{1}{2}\int_{\alpha}^{\beta} [r(\theta)]^2 d\theta$$



$$A = 2\left[\frac{1}{2}\int_0^{\frac{\pi}{3}} (2\cos\theta)^2 d\theta - \frac{1}{2}\int_0^{\frac{\pi}{3}} 1^2 d\theta\right] = \int_0^{\frac{\pi}{3}} 4\cos^2\theta - 1\, d\theta$$