MTH 264, Quiz #14, Fall 2023 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the slope of the tangent line and concavity (if possible) for the graph defined by $x = 2 + \sec t, y = 1 + 2\tan t$ at the point $t = \frac{\pi}{6}$.

$\frac{dx}{dt} = \sec t \tan t \quad \frac{dy}{dt} = 2\sec^2 t \quad \frac{dy}{dx} = \frac{2\sec^2 t}{\sec t \tan t} = \frac{2\sec t}{\tan t} = \frac{2\cos t}{\sin t}\cdot\frac{1}{\cos t} = 2\csc t = m$

$= 4$

$\frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt} = \frac{-\csc t \cot t}{\sec t \tan t} = -\frac{\cos t \cot t}{\sin t \tan t} = -\frac{\cot^2 t}{\tan t} = -\cot^3 t < 0$ Concave down

2. Set up an integral to find the arclength of the curve $x = \arctan t, y = \frac{1}{2}\ln\left(\sqrt{1 - t^2}\right)$ on the interval $\left[0, \frac{1}{2}\right]$. You do not need to evaluate the integral.

$\frac{1}{2}\cdot\frac{1}{1-t^2}\cdot -2t = \frac{-1}{1-t^2}$

$S = \int_0^{1/2} \sqrt{\left(\frac{1}{1+t^2}\right)^2 + \left(\frac{-1}{1-t^2}\right)^2}\,dt = \int_0^{1/2} \sqrt{\frac{(1-t^2)^2 + (1+t^2)^2}{(1+t^2)^2(1-t^2)^2}}\,dt = \int_0^{1/2} \sqrt{\frac{1-2t^2+t^4+1+2t^2+t^4}{(1+t^2)^2(1-t^2)^2}}\,dt =$

$\int_0^{1/2} \sqrt{\frac{2+2t^4}{(1-t^4)^2}}\,dt$

3. Set up an integral to find the area of the surface generated by revolving $x = \frac{1}{3}t^3, y = t + 1$ on the interval $[1,2]$ around the y-axis. You do not need to evaluate it. $\frac{dx}{dt} = t^2 \quad \frac{dy}{dt} = 1$

$S = 2\pi\int_1^2 \left(\frac{1}{3}t^3\right)\sqrt{t^4+1}\,dt \qquad u = t^4+1$

$du = 4t^3\,dt$

$2\pi\int \frac{1}{12}u^{1/2}\,du = \frac{1}{12}\cdot\frac{2}{3}u^{3/2}\Big| \qquad \frac{1}{4}du = t^3\,dt$

$2\pi\left[\frac{1}{18}(17)^{3/2} - \frac{1}{18}(2)^{3/2}\right] = \frac{\pi}{9}\left(17^{3/2} - 2^{3/2}\right)$

4. Convert the equation into the indicated coordinate system.
   a. Convert $xy = 4$ into polar coordinates. $r\cos\theta\, r\sin\theta = 4 \rightarrow r^2 = \frac{4}{\cos\theta\sin\theta}$
   b. Convert $\theta = \frac{5\pi}{6}$ into rectangular coordinates.

$\tan(5\pi/6) = \frac{-1}{\sqrt{3}}$

$y = -\frac{1}{\sqrt{3}}x$