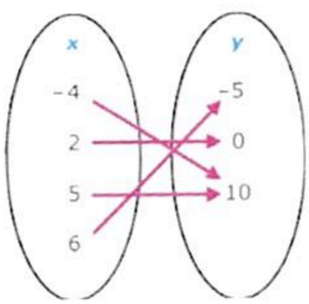
64
86
150

MTH 161, Exam #1, Fall 2024 Name KEY

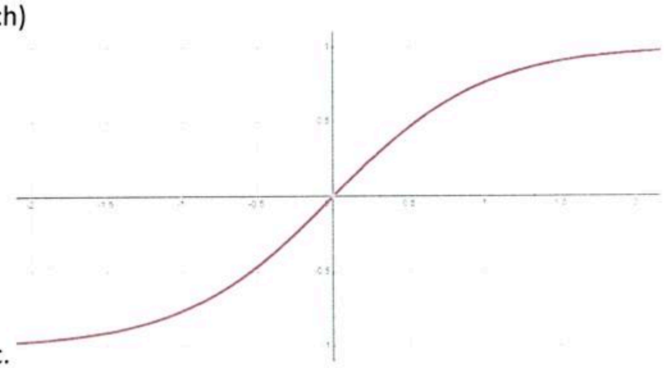
Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

Part 1: These questions you will submit answers to in Canvas. Show all work and submit the work with Part 2 of the exam. But you must submit the answers in Canvas to receive credit. Each question/answer will be listed separately. The Canvas question will refer to the number/part to indicate where you should submit which answer. The questions will appear in order (in case there is an inadvertent typo). Correct answers will receive full credit with or without work in this section, but if you don't submit work and clearly label your answers, you won't be able to challenge any scoring decisions for making an error of any kind.

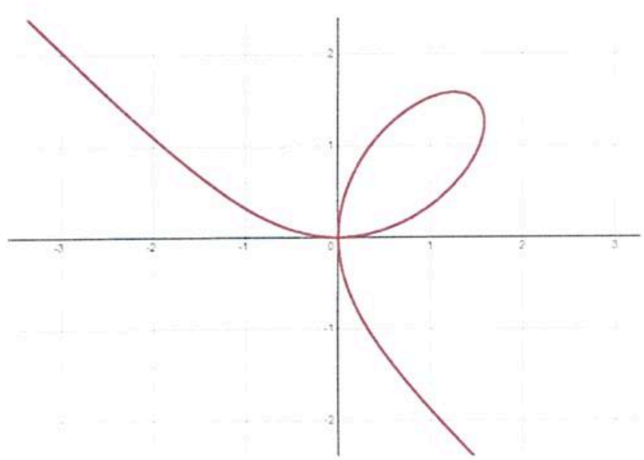
1. For each of the following relations, determine i) if the relation is a function, ii) if it is a function, is its inverse also a function. (4 points each)



a.



c.



b.

A. function
inverse is also a function
B. function
inverse is also a function
C. not a function

2. What is the average rate of change of the function $f(x) = 1 - \frac{1}{6}x^3$ on the interval $[-2,3]$? (6 points)

$f(-2) = 7/3 \quad f(3) = -7/2$

$$\frac{-7/2 - 7/3}{3-(-2)} = \frac{-35/6}{5} = -7/6$$

3. Consider the points $(-1,-4), (3,-2)$. Find: (4 points each)
   a. The distance between the two points

$$\sqrt{(-1-3)^2+(-4+2)^2} = \sqrt{(-4)^2+(-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

   b. The midpoint of the line segment connecting the points.

$$\left(\frac{-1+3}{2}, \frac{-4+(-2)}{2}\right) = \left(\frac{2}{2}, \frac{-6}{2}\right) = (1,-3)$$

   c. What is the slope of the line connecting the two points?

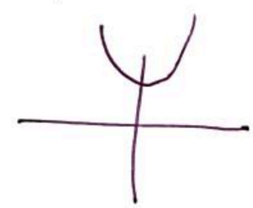
$$\frac{-2-(-4)}{3-(-1)} = \frac{-2+4}{3+1} = \frac{2}{4} = \frac{1}{2}$$

4. Let $P(x,y)$ be a point on the graph of $y = 2x^2 - 5$. Express the distance d from P to the point $(1,0)$, as a function of the point's x-coordinate. Find the minimum distance graphically. (6 points)

(x, y)

$$d = \sqrt{(x-1)^2 + (2x^2-5-0)^2} = \sqrt{x^2-2x+1+4x^4-20x^2+25}$$

$$= \sqrt{4x^4-19x^2+26}$$



min at $(0,5)$

5. The endpoints of a circle's diameter are $(3,-2)$ and $(-6,-6)$. Find the center of the circle, its radius, and equation in standard form. (8 points)

$$\left(\frac{3+(-6)}{2}, \frac{-2+(-6)}{2}\right) = \left(\frac{3}{2}, \frac{-8}{2}\right) = \left(\frac{3}{2}, -4\right)$$

center

$$r = d = \sqrt{\left(\frac{3}{2}-3\right)^2 + (-4+2)^2} = \sqrt{\left(\frac{3}{2}\right)^2 + (-2)^2} = \sqrt{\frac{9}{4}+4} = \sqrt{\frac{25}{4}} = \frac{5}{2}$$

$$\left(x-\frac{3}{2}\right)^2 + (y+4)^2 = \frac{25}{4}$$

6. Find the vertex, the y-intercept and any zeros (real and complex) of the function $f(x) = -\frac{1}{2}(x+3)^2 + 4$. Use that information to sketch the graph. [If the zeros are complex, you may need to plot additional points by hand; if so, use the symmetry of the graph.] (8 points)

vertex $(-3, 4)$

$-\frac{1}{2}(x^2+6x+9)+4 =$

$-\frac{1}{2}x^2 - 3x - \frac{9}{2} + 4 = -\frac{1}{2}x^2 - 3x - \frac{1}{2}$

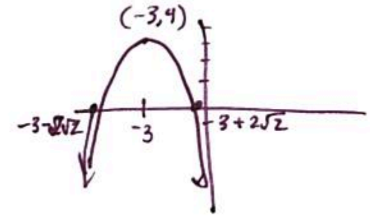
$-\frac{1}{2}(0+3)^2+4 =$

$-\frac{1}{2}$ (y-int)

$-\frac{1}{2}x^2 - 3x - \frac{1}{2} = 0$

$x^2 + 6x + 1 = 0$

$x = \frac{-6 \pm \sqrt{36-4(1)(1)}}{2} = \frac{-6 \pm \sqrt{32}}{2} = \frac{-6 \pm 4\sqrt{2}}{2} = -3 \pm 2\sqrt{2}$



7. For each of the following functions, determine:
   i) any relative extrema (relative maxima or minima),
   ii) symmetry (even, odd or neither).

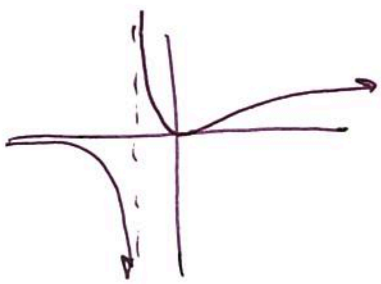
   [Hint: it's helpful to sketch the graph.] (6 points each)

a. $f(x) = \frac{x^2}{x^3+1}$

no symmetry

$\frac{(-x)^2}{(-x)^3+1} = \frac{x^2}{-x^3+1} = \frac{-x^2}{x^3-1}$

none



relative min @ $(0,0)$

relative max @ $\approx (1.26, 0.529)$

b. $f(x) = |\sqrt{x^2+4} - 8|$

even symmetry

$|\sqrt{(-x)^2+4} - 8| = |\sqrt{x^2+4} - 8|$

even

relative max at $(0, 6)$

relative mins @ $(\pm 2\sqrt{15}, 0)$

$\sqrt{x^2+4} - 8 = 0$

$\sqrt{x^2+4} = 8$

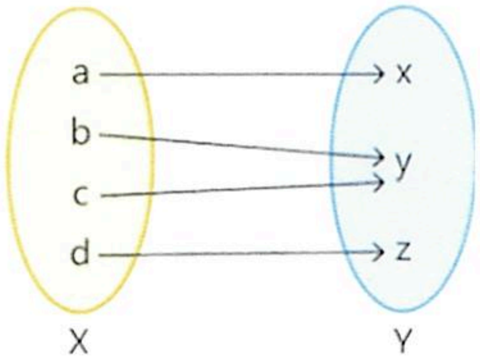
$x^2 + 4 = 64$

$x^2 = 60$

$x = \pm\sqrt{60} = \pm 2\sqrt{15}$

Part 2: In this section you will record your answers on paper along with your work. After scanning, submit them to a Canvas dropbox as directed. These questions will be graded by hand.

8. State the domain and range of the relation in set notation. State the domain and range of the inverse relation. Clearly indicate which is which. (4 points)



original

D: $\{a, b, c, d\}$

R: $\{x, y, z\}$

inverse

D: $\{x, y, z\}$

R: $\{a, b, c, d\}$

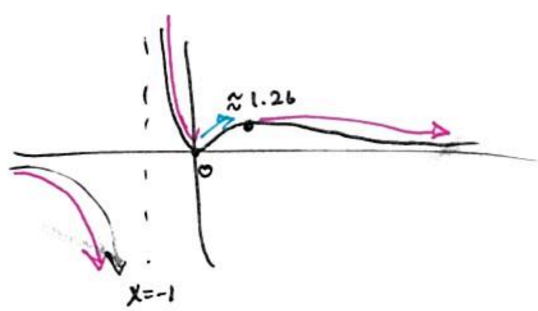
9. For each of the following functions, determine:
   iii) any intervals on which the function is increasing,
   iv) intervals on which the function is decreasing,
   v) intervals on which the function is constant,
   [Hint: it's helpful to sketch the graph.] (6 points each)

   c. $f(x) = \frac{x^2}{x^3+1}$

decreasing $(-\infty, -1) \cup (-1, 0) \cup (1.26, \infty)$

increasing $(0, 1.26)$ (approx)

never constant

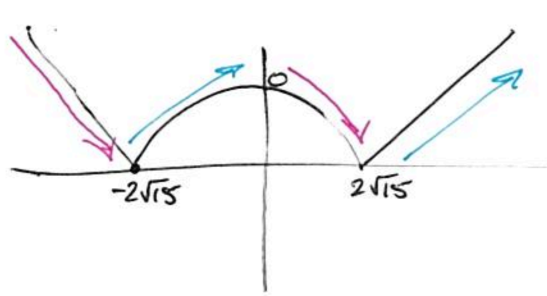


   d. $f(x) = \left|\sqrt{x^2+4} - 8\right|$

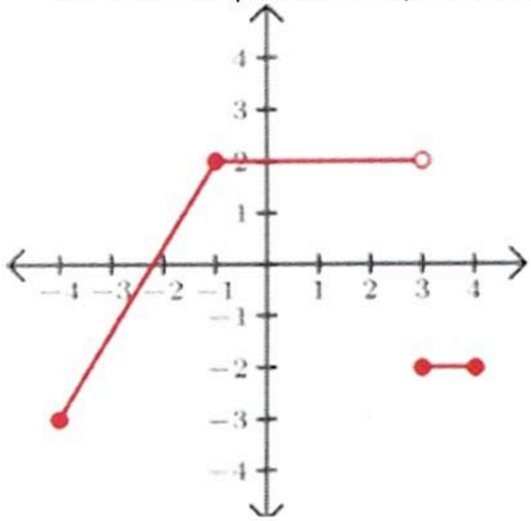
decreasing $(-\infty, -2\sqrt{15}) \cup (0, 2\sqrt{15})$

increasing $(-2\sqrt{15}, 0) \cup (2\sqrt{15}, \infty)$

never constant



10. Write an equation of the piecewise graph shown. (8 points)



$(-4,-3)$
$(-1,2)$

$m = \dfrac{-3-2}{-4+1} = \dfrac{-5}{-3} = \dfrac{5}{3}$

$y-2 = \frac{5}{3}(x+1)$

$y-2 = \frac{5}{3}x + \frac{5}{3}$
$\quad +2 \qquad\quad +2$

$y = \frac{5}{3}x + \frac{11}{3}$

$$f(x) = \begin{cases} \frac{5}{3}x + \frac{11}{3} & -4 \le x \le -1 \\ 2 & -1 < x < 3 \\ -2 & 3 \le x \le 4 \end{cases}$$

11. Find $\frac{f(x+h)-f(x)}{h}$ for $f(x) = -2x^2 + 3x - 5$. (8 points)

$$\frac{-2(x+h)^2 + 3(x+h) - 5 - (-2x^2+3x-5)}{h} =$$

$$\frac{-2(x^2+2xh+h^2) + 3x + 3h - 5 + 2x^2 - 3x + 5}{h} =$$

$$\frac{-\cancel{2x^2} - 4xh - 2h^2 + \cancel{3x} + 3h - \cancel{5} + \cancel{2x^2} - \cancel{3x} + \cancel{5}}{h} =$$

$$\frac{-4xh - 2h^2 + 3h}{h} = \frac{\cancel{h}(-4x-2h+3)}{\cancel{h}} =$$

$$-4x - 2h + 3$$

12. Find an equation of the line with the following properties: (5 points each)

a. Passing through the points $(3,2)$ and $(5,-3)$.

$$m = \frac{-3-2}{5-3} = \frac{-5}{2} = -\frac{5}{2}$$

$$y-2 = -\frac{5}{2}(x-3) \rightarrow \underset{+2}{y-2} = \underset{+2}{-\frac{5}{2}x + \frac{15}{2}} \rightarrow y = -\frac{5}{2}x + \frac{19}{2}$$

b. Perpendicular to the line $2x + 5y = 10$ and passing through $(-4,2)$.

$$\frac{5y}{5} = \frac{-2x+10}{5} \qquad m_2 = \frac{5}{2}$$

$$y = -\frac{2}{5}x + 2$$

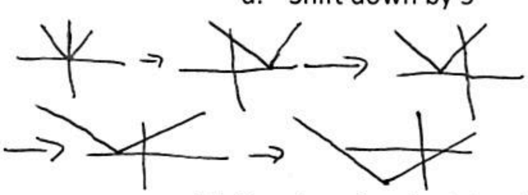
$$y-2 = \frac{5}{2}(x+4) \rightarrow \underset{+2}{y-2} = \underset{+2}{\frac{5}{2}x + 10} \rightarrow y = \frac{5}{2}x + 12$$

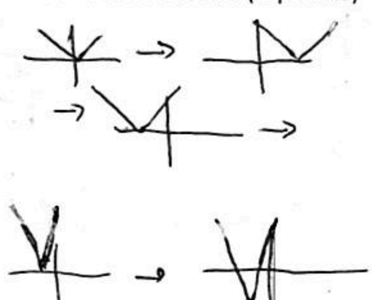
c. Parallel to $x = -4$ and passing through $(-3,2)$.

$$x = -3$$

13. If $f(x) = |x|$, write the function that has the following transformations applied in the given order. Your final function should contain all the transformations. (4 points)

a. Shift right 7 units
b. Reflect over the y-axis
c. Compress by a factor of 2
d. Shift down by 5





$|x-7|$

$|-x-7|$

$\frac{1}{2}|-x-7|$

$\frac{1}{2}|-x-7|-5$

or

$|-2x-7|-5$

14. Create a sign chart to solve the quadratic inequality $2x^2 + 9x - 5 \leq 0$. Write the solution in interval notation. (5 points)

$$(2x-1)(x+5) = 0$$

$$x = 1/2, \quad x = -5$$

$+ \qquad - \qquad +$

$-5 \qquad 1/2$

$$[-5, 1/2]$$

15. Shown is the function $f(x)$. Sketch the graph of $3f(-x+2)-2$. (10 points)

$-(x-2)$

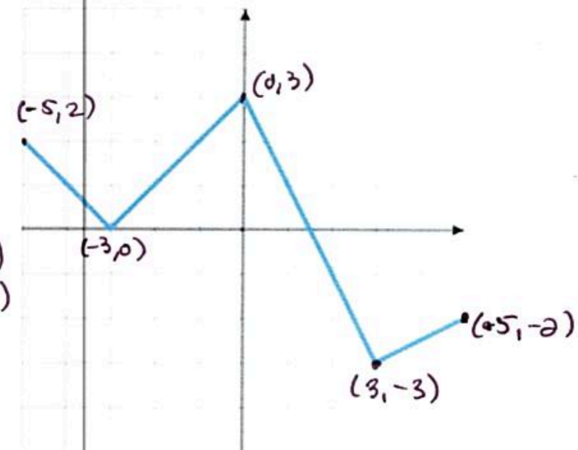
$(-5,2) \to (-3,2) \to (3,2) \to (3,6) \to (3,4)$

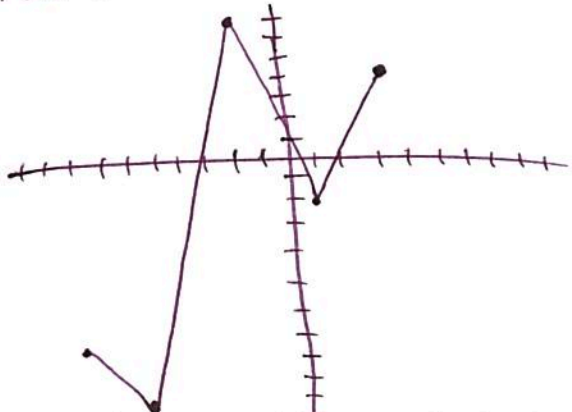
$(-3,0) \to (-1,0) \to (1,0) \to (1,0) \to (1,-2)$

$(0,3) \to (2,3) \to (-2,3) \to (-2,9) \to (-2,7)$

$(3,-3) \to (5,-3) \to (-5,-3) \to (-5,-9) \to (-5,-11)$

$(5,-2) \to (7,-2) \to (-7,-2) \to (-7,-6) \to (-7,-8)$





16. Create a linear regression equation for the data shown below. Write the equation. Interpret the slope in the context of the problem. (10 points)

i	Avg. Temperature (°C)	Avg. Rainy Days
1	0.2	12.1
2	1.6	11.1
3	5.7	10.7
4	11.3	11
5	16.1	12.1
6	20.1	8.4
7	23.5	3.4
8	23.4	2.6
9	18.8	4
10	12.9	6.8
11	7.1	8
12	2.4	11.6

$y = -0.32255x + 12.32975$

for every degree increase in (Celsius) average temperature, one can expect on average 0.32 fewer rainy days.

17. Given $f(x) = 2x^2 - 4$, $g(x) = \sqrt{x+3}$, $h(x) = \frac{1}{x+1}$, find the following functions and state the domain. (5 points each)

a. $(f + h)(x)$

$(f+h)(x) = 2x^2 - 4 + \frac{1}{x+1}$

D: $(-\infty, -1) \cup (-1, \infty)$

b. $\left(\frac{f}{g}\right)(x)$

$\left(\frac{f}{g}\right)(x) = \frac{2x^2-4}{\sqrt{x+3}}$

D: $(-3, \infty)$

no endpoint makes denom = 0

c. $(g \circ h)(x)$

$g(h(x)) = \sqrt{\frac{1}{x+1} + 3}$

$\frac{1}{x+1} + 3 \geq 0$ $\quad \frac{1 + 3(x+1)}{x+1} \geq 0 = \frac{3x+4}{x+1} \geq 0$ $\quad x = -1,\ x = -4/3$

D: $(-\infty, -4/3) \cup (-1, \infty)$

18. Find the inverse of $f(x) = \frac{x+2}{2x-1}$. Sketch the graph and its inverse on the same graph. Describe the symmetry you see. (10 points)

$x = \frac{y+2}{2y-1}$

$x(2y-1) = y+2$

$2xy - x = y + 2$

$2xy - y = x + 2$

$y(2x-1) = x+2$

$y = \frac{x+2}{2x-1}$

The function is it's own inverse

x- int = -2

y- int = -2