MTH 161, Homework #6, Fall 2024 Name ____

Instructions: Write your work up neatly and attach to this page. Record your final answers (only) directly on this page if they are short; if too long indicate which page of the work the answer is on and mark it clearly. Use exact values unless specifically asked to round.

1. Solve the system of equations using elementary row operations on the augmented matrix for the system. State your solution in vector form, or state that the solution is inconsistent (no solution). If the solution is consistent (has a solution), state whether it is dependent (infinitely many solutions) or independent (one solution). If the system is dependent, write the solution in parametric form. (For four and five variables systems, you may use technology; two and three variable systems should be solved by hand.)

a. $\begin{cases} 3x_1 + 6x_2 = -3 \\ 5x_1 + 7x_2 = 10 \end{cases}$

b. $\begin{cases} 2x_1 \qquad\qquad\qquad - 4x_4 = -10 \\ \qquad 3x_2 + 3x_3 \qquad = 0 \\ \qquad\qquad x_3 + 4x_4 = -1 \\ -3x_1 + 2x_2 + 3x_3 + x_4 = 5 \end{cases}$

c. $\begin{cases} x_1 - 2x_2 - 3x_3 = 0 \\ \qquad x_2 + 2x_3 = 0 \\ 2x_1 - 4x_2 + 9x_3 = 0 \end{cases}$

d. $\begin{cases} 2x_1 \qquad - 6x_3 = -8 \\ \qquad x_2 + 2x_3 = 3 \\ 3x_1 + 6x_2 - 2x_3 = -4 \end{cases}$

e. $\begin{cases} 2x_1 - 5x_2 + 8x_3 = 0 \\ -3x_1 - 4x_2 + 2x_3 = 0 \end{cases}$

f. $\begin{cases} 2x_1 \qquad\qquad\qquad - 4x_4 + x_5 = 0 \\ \qquad 3x_2 + 3x_3 \qquad - x_5 = 0 \\ \qquad\qquad x_3 + 4x_4 + 6x_5 = 0 \\ -3x_1 + 2x_2 + 3x_3 + x_4 - 2x_5 = 0 \end{cases}$

2. Solve the systems below by substituting $a = \frac{1}{x}, b = \frac{1}{y}, c = \frac{1}{z}$. Be sure to state your final solution in the original x, y, z variables.

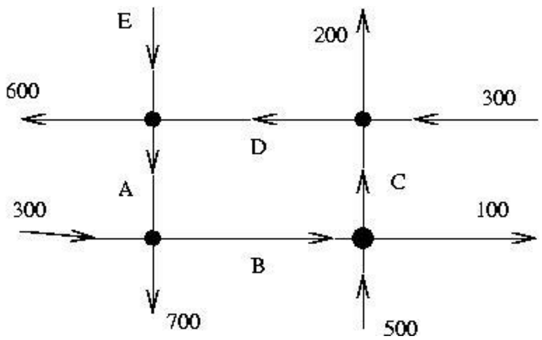
a. $\begin{cases} \frac{12}{x} - \frac{12}{y} = 7 \\ \frac{3}{x} + \frac{4}{y} = 0 \end{cases}$

b. $\begin{cases} \frac{2}{x} + \frac{1}{y} - \frac{2}{z} = 5 \\ \frac{3}{x} - \frac{4}{y} \qquad = -1 \\ \frac{2}{x} + \frac{1}{y} + \frac{3}{z} = 0 \end{cases}$

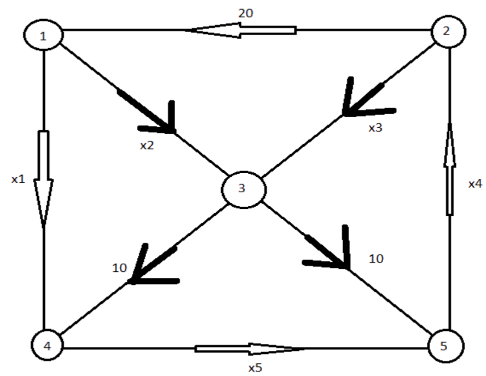
3. Find an appropriate interpolating polynomial $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots a_nx^n$ or other indicated equation for the given data. Recall that you are creating a system of equations that can be solved for the unknown coefficients. Use substitutions as appropriate.
   a. $(1,7), (2,17), (3,31), (4,65)$, cubic
   b. $(-2,28), (-1,0), (0,-6), (1,-8), (2,0)$, quartic

4. For each of the traffic networks below, set up a system of equations to solve it. Solve the system with technology. Is there a single solution?

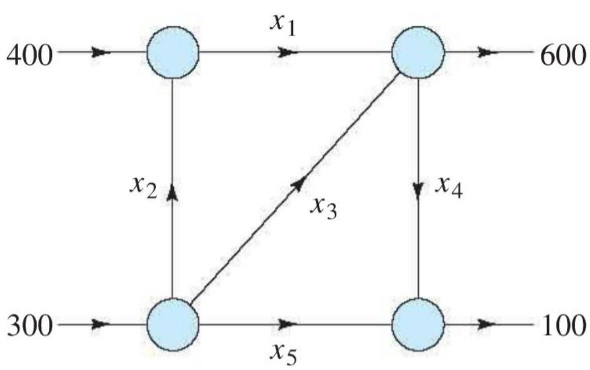
a.



c.



b.



5. Consider the following matrices.

$$A=\begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}, B=\begin{bmatrix} 9 & 3 \\ 1 & 0 \end{bmatrix}, C=\begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix}, D=\begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}, E=\begin{bmatrix} 0 & -3 & 5 \\ 1 & -4 & 0 \\ -1 & 2 & -7 \end{bmatrix}$$

$$F=\begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & 4 & -1 & 5 \end{bmatrix}, G=\begin{bmatrix} 6 & -7 \\ 11 & -5 \\ 2 & 3 \end{bmatrix}, H=\begin{bmatrix} 1 & 0 & -2 \end{bmatrix}, J=\begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

a. Add

i. A+B
ii. 2B-3C

b. Multiply

i. 4A
ii. -5H

c. Multiply. If the multiplication is not possible, explain why not.

i. AB
ii. DE
iii. GC
iv. EH
v. BA
vi. BF
vii. DG
viii. BJ

d. Find the inverse, if possible. If not, explain why not.

i. A
ii. B
iv. D
v. E

iii. C

e. Find the determinant of the matrix.

i. A

ii. B

iii. C

iv. D

v. E

6. For problems 1a, 1b, 1c, 2a, 2b use Cramer's Rule to solve the system. Verify that the solutions are the same as with row-reducing (assuming Cramer's Rule to can be used). If the system can't be solved by this method, explain why not.

7. For problems 1a, 1b, 1c, 2a, 2b use the method of inverses to solve the system. Verify that the solutions are the same as the row-reducing method (assuming inverses can be used). If the system can't be solved by this method, explain why not.

8. Decompose the following rational expressions.

a. $\frac{4}{(x+1)(x+2)}$

b. $\frac{x-1}{(x^2+1)(x+4)}$

c. $\frac{x^2+2x-7}{(x^2+4)(x-1)}$

d. $\frac{1}{(x-2)^2(x+3)}$

9. Set up but do not solve the following partial fraction decompositions.

a. $\frac{x^4+x^2-2}{(x^2+1)^2(x-1)(x-2)^2(x^2+2x+5)}$

b. $\frac{x^3-6}{(x^2+4)(x+1)^3(x-2)}$

c. $\frac{x^2+6x+11}{(x^2-4)^2(x^2+1)(x-6)}$