10/10/2024

Polynomial and Rational Inequalities
Problems of Variation

Reminder: no class on Tuesday, review for Exam #2 on Thursday and Exam #2 a week from Tuesday.

To solve polynomial and rational inequalities: get everything on one side and compare a single function to 0.

Find the solution to the rational inequality and write it in interval notation:

$$\frac{-x^3+4x}{x^2-9} \le 4x$$

$$\frac{-x^3+4x}{x^2-9} - 4x \le 0$$

For a rational inequality, try to find a common denominator. You cannot eliminate the denominator at any point when solving an inequality.

$$\frac{-x^3+4x}{x^2-9} - 4x\left(\frac{x^2-9}{x^2-9}\right) \le 0$$

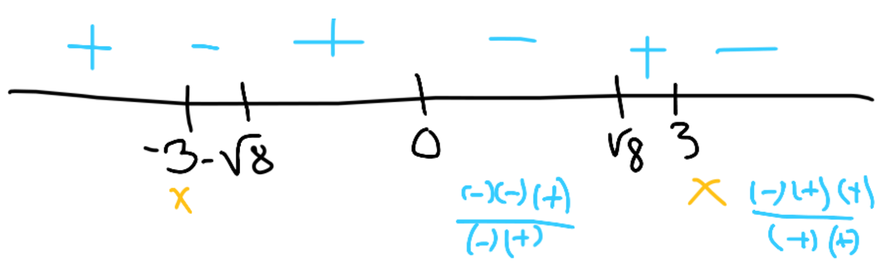
$$\frac{-x^3+4x-4x(x^2-9)}{x^2-9} \le 0$$

$$\frac{-x^3+4x-4x^3+36x}{x^2-9} \le 0$$

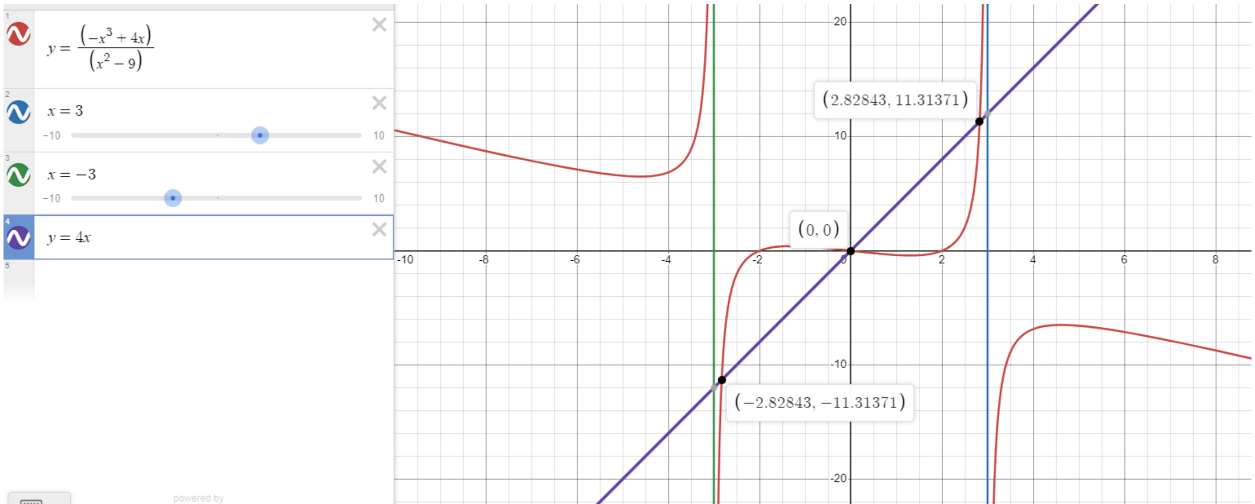
$$\frac{-5x^3+40x}{x^2-9} \le 0$$

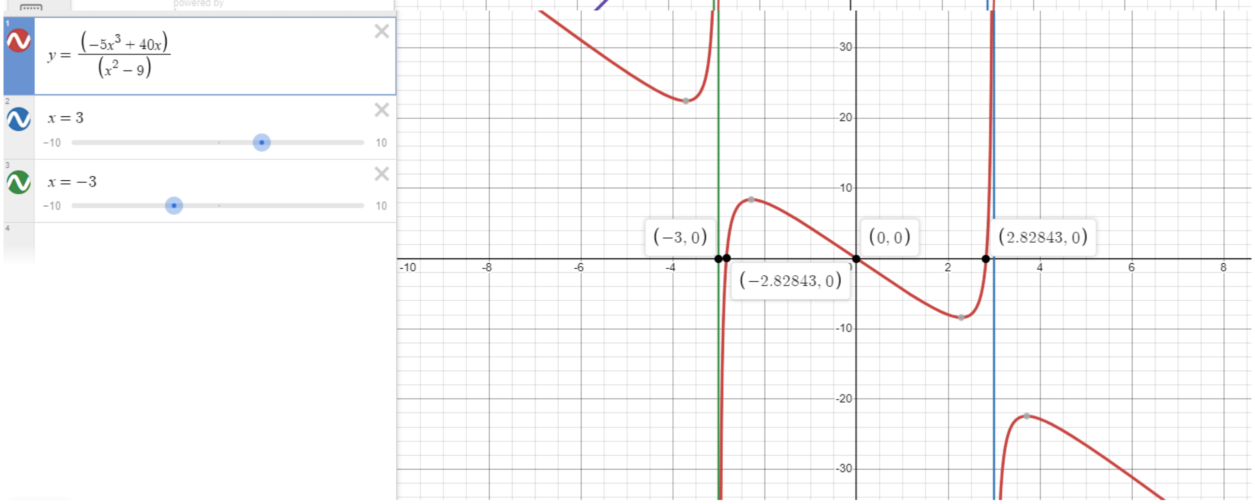
$$\frac{(-5x)(x^2-8)}{x^2-9} \le 0$$

$$\frac{(-5x)(x-\sqrt{8})(x+\sqrt{8})}{(x-3)(x+3)} \le 0$$



$$\left(-3,-2\sqrt{2}\right] \cup \left[0,2\sqrt{2}\right] \cup (3,\infty)$$



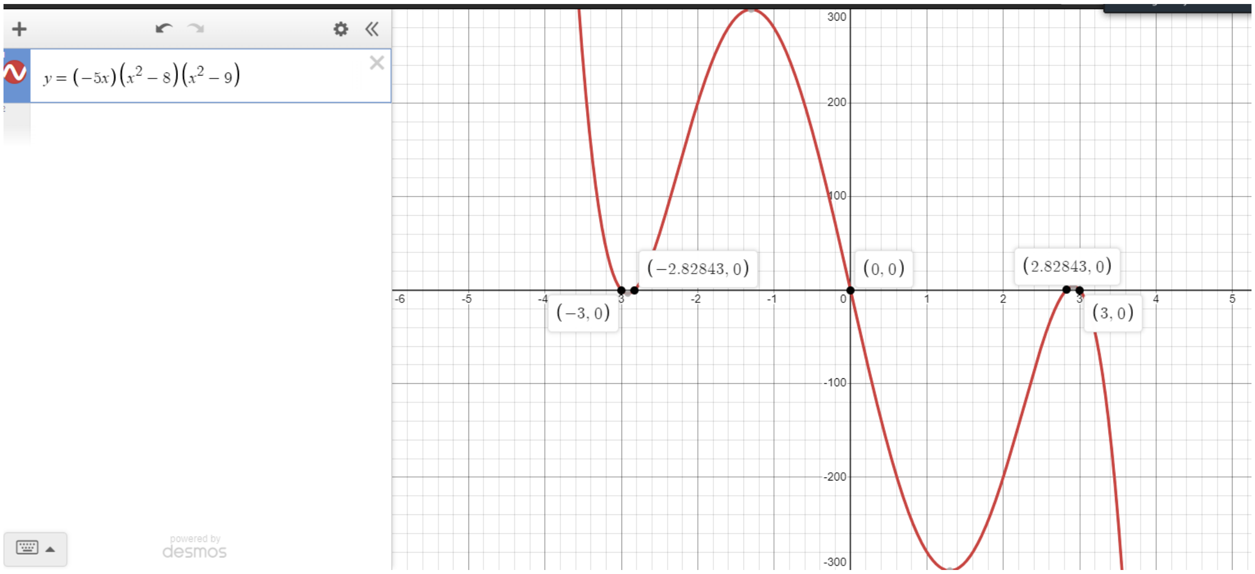


How do rational inequalities relate to polynomial inequalities?
Compare

$$(-5x)(x^2-8)(x^2-9) \leq 0$$

Vs.

$$\frac{(-5x)(x^2-8)}{x^2-9} \leq 0$$



All the sign change points are identical to the rational function. The solution differs only where the factors appeared in the denominator:

$$[-3,-2\sqrt{2}]\cup[0,2\sqrt{2}]\cup[3,\infty)$$

Problem of Variation
Direct Variation, Indirect Variation and Joint Variation
Direct and Indirect Variation only involve one variable that depend on one other variable.
Joint variation has one variable that depends on more than one variable

The first sentence will describe the relationship between the variables. The second sentence gives you information about all the variables in the problem so that you can solve of the constant of variation (proportionality constant). Third sentence will give you information about the variables in the formula except for one, and they ask you to solve for the missing variable value.

Suppose that y varies directly with x. When $y=6, x=18$. What is the value of y when $x=24$?

Direct variation:

$$y=kx$$

Inverse variation:

$$y=\frac{k}{x}$$

Joint variation:

$$y=kxz$$
$$y=\frac{kx}{z}$$

For this problem:

$$y = kx$$

$$6 = k(18)$$

$$k = \frac{6}{18} = \frac{1}{3}$$

$$y = \frac{1}{3}x$$

$$y = \frac{1}{3}(24) = 8$$

Suppose that the temperature in a room is inversely proportional to the humidity. If the temperature in the room is 70°F, the humidity in the room is 25%. Find the temperature if the humidity rises to 30%.

$$T = \frac{k}{H}$$

$$70 = \frac{k}{0.25}$$

$$k = 17.5$$

$$T = \frac{17.5}{0.3} = 58.3\text{°F}$$

Suppose that y varies jointly with x and inversely with z. When $x = 3, z = 4, y = 60$. Find the value of y when $x = 5\ and\ z = 2$.

$$y = \frac{kx}{z}$$

$$60 = \frac{k(3)}{4}$$

$$k = \frac{(60)(4)}{3} = 80$$

$$y = \frac{80x}{z}$$

$$y = \frac{80(5)}{2} = 200$$

This the last thing that is on Exam #2 (10/22)
Chapters 3 and 4 in the online textbook.