9/24/2024

Go over exam #1
Polynomial Functions

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

n is a non-negative integer, a_i are real

example.

$$p(x) = 4\ (degree = 0)$$
$$p(x) = 3x - 2\ (degree = 1)$$
$$p(x) = x^2 - 5x + 11\ (degree = 2)$$
$$p(x) = x^7 - x^3 + 1\ (degree = 7)$$

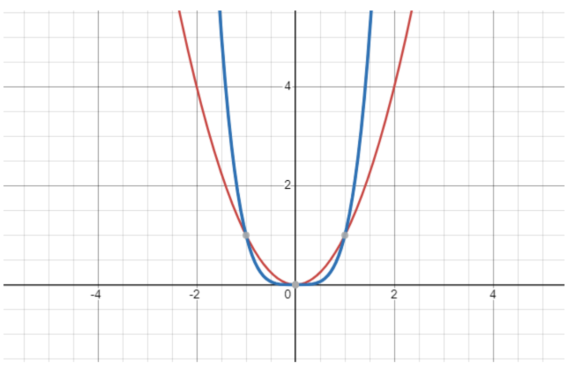
Descending order is standard.

Leading coefficient: is the coefficient of the highest degree term.

End behavior of polynomials:

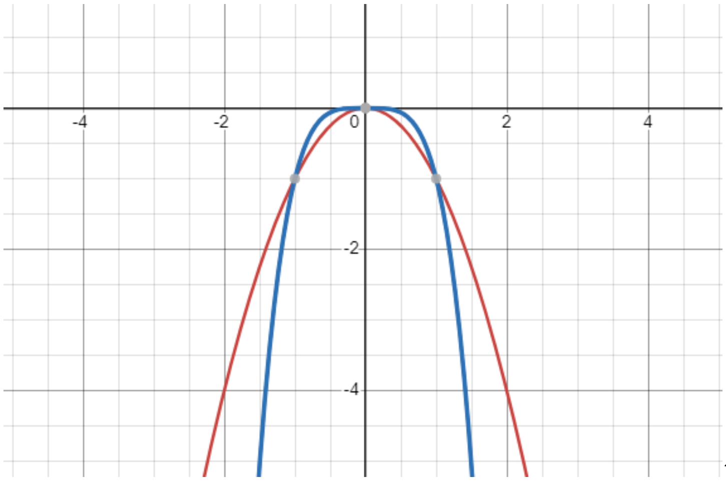
Is going to depend on the degree of the polynomials: even degrees, and odd degrees.

Even polynomials:



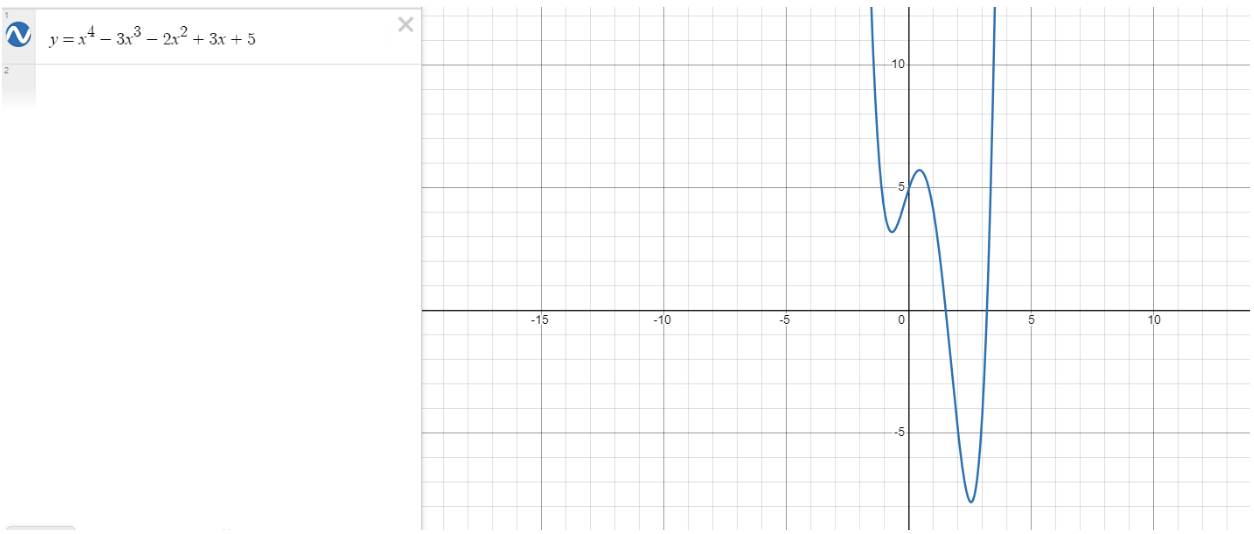
$red: y = x^2, blue: y = x^4$

Start on the left at infinity (in y) and end at infinity (in y) on the right, if the leading coefficient is positive. If the leading coefficient is negative, then the graph opens down, and goes to negative infinity on both sides.

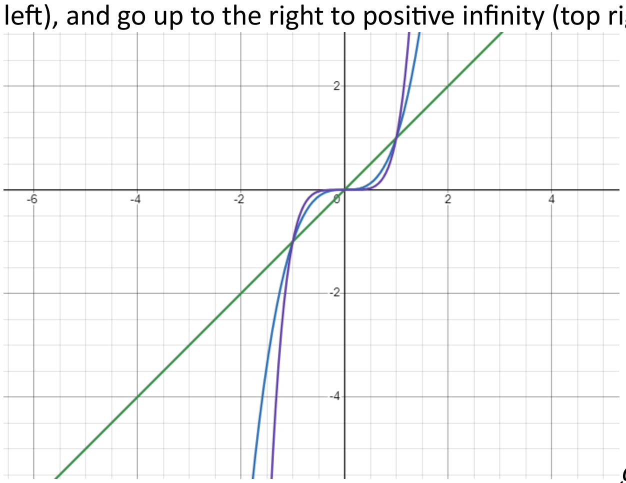


$red: y = -x^2, blue: y = -x^4$

These functions will have either a global max (if they open down) or a minimum (if they open up)

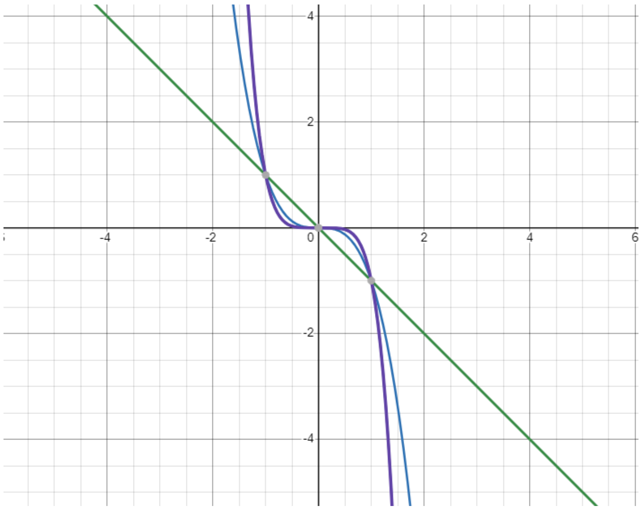


For odd degree polynomials: positive leading coefficients will start on the left at negative infinity (bottom left), and go up to the right to positive infinity (top right)



$green: y = x, blue: y = x^3, purple: y = x^5$

If the leading coefficient is negative, then they start on the top left (positive infinity) and end on the bottom right (negative infinity)



$green: y = -x, blue: y = -x^3, purple: y = -x^5$

Intermediate Value Theorem: if f(a)<0 and f(b)>0 (or vice versa), then there is a zero (x-intercept) of the polynomial between those values (on the interval (a,b)).

Suppose a polynomial is in factored form (linear factors or unfactorable quadratics)

$$p(x) = (x + 1)(x - 3)^2(x^2 + 4)$$

$$x + 1 = 0, x = -1$$
$$x - 3 = 0, x = 3$$
$$x^2 + 4 = 0\ never$$

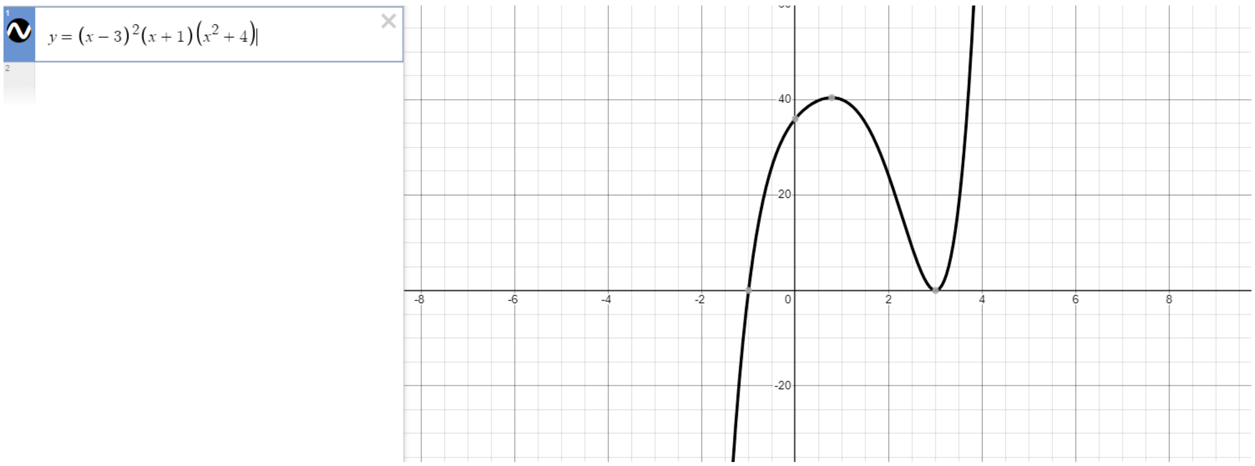
There are two zeros (x-intercepts) of the polynomial, with one at x=-1, and one at x=3.

What is the highest degree of the polynomial? Fifth degree polynomial with positive leading coefficient.

Multiplicity of a root (or a zero) is determined by the number of times the linear factor appears in the factored form.
x=-1 has a multiplicity of 1
and x=3 has a multiplicity of 2

The multiplicity is going to control what the polynomial does when it gets to 0 (in y).

If the multiplicity is odd, then the polynomial crosses the x-axis at that zero. And if the multiplicity is even, the polynomial touches the axis but does not cross it.



$y = (x-3)^2(x+1)(x^2+4)$
40
20
-20
-8
-6
-4
-2
0
2
4
6
8