Math 1149 Domain & Range of Trig Functions & Inverses Key

a.

$F(x) = \sin(2x)$

D: $(-\infty, \infty)$ (restricted $\left[\frac{-\pi/2}{2}, \frac{+\pi/2}{2}\right] = [-\pi/4, \pi/4]$)

R: $[-1, 1]$

$y = \sin 2x \Rightarrow x = \sin(2y) \Rightarrow \sin^{-1}(x) = 2y \Rightarrow$

$F^{-1}(x) = \frac{1}{2}\sin^{-1}(x)$ or $\frac{1}{2}\arcsin(x) = F^{-1}(x)$

D: $[-1, 1]$

R: $[-\pi/4, \pi/4]$

b.

$G(x) = 3\cos x$

D: $(-\infty, \infty)$ (restricted $[0, \pi]$)

R: $[-3, 3]$

$y = 3\cos x \Rightarrow x = 3\cos y \Rightarrow \frac{x}{3} = \cos y \Rightarrow y = \cos^{-1}\left(\frac{x}{3}\right)$

$G^{-1}(x) = \cos^{-1}\left(\frac{x}{3}\right) = \arccos\left(\frac{x}{3}\right)$

D: $[-3, 3]$

R: $[0, \pi]$

c.

$H(x) = 4\sin(x - \pi)$

D: $(-\infty, \infty)$ (restricted $[-\pi/2 + \pi, \pi/2 + \pi] = [\pi/2, 3\pi/2]$)

R: $[-4, 4]$

$y = 4\sin(x-\pi) \Rightarrow x = 4\sin(y-\pi) \Rightarrow \frac{x}{4} = \sin(y-\pi) \Rightarrow$

$\sin^{-1}\left(\frac{x}{4}\right) = y - \pi \Rightarrow \sin^{-1}\left(\frac{x}{4}\right) + \pi = y$

$H^{-1}(x) = \sin^{-1}\left(\frac{x}{4}\right) + \pi = \arcsin\left(\frac{x}{4}\right) + \pi$

c. Cont'd

D: $[-4, 4]$ R: $[\pi/2, 3\pi/2]$

d. $J(x) = 5\tan(x)$

D: $x \neq \frac{(2k+1)\pi}{2}$ (restricted $(-\pi/2, \pi/2)$)

R: $(-\infty, \infty)$

$y = 5\tan x \Rightarrow x = 5\tan y \Rightarrow \frac{x}{5} = \tan y \Rightarrow \tan^{-1}(\frac{x}{5}) = y$

$J^{-1}(x) = \tan^{-1}(\frac{x}{5}) = \arctan(\frac{x}{5})$

D: $(-\infty, \infty)$ R: $(-\pi/2, \pi/2)$

e. $K(x) = -3\cot(2x)$

D: $x \neq \frac{k\pi}{2}$ (restricted $(0/2, \pi/2) = (0, \pi/2)$)

R: $(-\infty, \infty)$

$y = -3\cot(2x) \Rightarrow x = -3\cot(2y) \Rightarrow -\frac{x}{3} = \cot(2y) \Rightarrow$

$\cot^{-1}(-\frac{x}{3}) = 2y \Rightarrow -\frac{1}{2}\cot^{-1}(\frac{x}{3}) = y$

$K^{-1}(x) = -\frac{1}{2}\cot^{-1}(\frac{x}{3}) = -\frac{1}{2}\operatorname{arccot}(\frac{x}{3})$

D: $(-\infty, \infty)$ R: $(0, \pi/2)$

f. $L(x) = 9\sec(x+1)$

D: $x \neq \frac{(2k+1)\pi}{2} - 1$ (restricted

R: $(-\infty, -9] \cup [9, \infty)$

$[0, \pi/2) \cup (\pi/2, \pi]$

$[-1, \pi/2 - 1) \cup (\pi/2 - 1, \pi - 1]$

$y = 9\sec(x+1) \Rightarrow x = 9\sec(y+1) \Rightarrow \frac{x}{9} = \sec(y+1) \Rightarrow$

$\sec^{-1}(\frac{x}{9}) = y+1 \Rightarrow \sec^{-1}(\frac{x}{9}) - 1 = y$

$L^{-1}(x) = \sec^{-1}(\frac{x}{9}) - 1 = \operatorname{arcsec}(\frac{x}{9}) - 1$

D: $(-\infty, -9] \cup [9, \infty)$ R:

$[-1, \pi/2 - 1) \cup (\pi/2 - 1, \pi - 1]$

③

g. $M(x) = -2\csc(4\pi x) + 1$

D: $x \neq \frac{k\pi}{4\pi} = \frac{k}{4}$

R: $(-\infty, -2+1] \cup [2+1, \infty) =$

$(-\infty, -1] \cup [3, \infty)$

(restricted $\left[-\frac{\pi/2}{4\pi}, 0\right) \cup \left(0, \frac{\pi/2}{4\pi}\right] =$

$[-1/8, 0) \cup (0, 1/8]$

$y = 2\csc(-4\pi x) + 1 \Rightarrow x = 2\csc(-4\pi y) + 1 \Rightarrow x - 1 = \csc(-4\pi y)$

$\Rightarrow \csc^{-1}(x-1) = -4\pi y \Rightarrow -\frac{1}{4\pi}\csc^{-1}(x-1)$

$M^{-1}(x) = -\frac{1}{4\pi}\csc^{-1}(x-1) = -\frac{1}{4\pi}\operatorname{arccsc}(x-1)$

D: $(-\infty, -1] \cup [3, \infty)$

R: $[-1/8, 0) \cup (0, 1/8]$

h. $N(x) = -\sin(\pi x) - 3$

D: $(-\infty, \infty)$

(restricted $\left[-\frac{\pi/2}{\pi}, \frac{\pi/2}{\pi}\right] = [-1/2, 1/2]$

R: $[-4, -2]$

$y = -\sin(\pi x) - 3 \Rightarrow x = \sin(-\pi y) - 3 \Rightarrow x + 3 = \sin(-\pi y)$

$\sin^{-1}(x+3) = -\pi y \Rightarrow -\frac{1}{\pi}\sin^{-1}(x+3) = y$

$N^{-1}(x) = -\frac{1}{\pi}\sin^{-1}(x+3) = -\frac{1}{\pi}\arcsin(x+3)$

D: $[-4, -2]$ R: $[-1/2, 1/2]$

i. $P(x) = 4\cos(-x - 3\pi/2) - 2$

D: $(-\infty, \infty)$ (restricted $[0 - 3\pi/2, \pi - 3\pi/2] = [-3\pi/2, -\pi/2]$)

R: $[-6, 2]$

$y = 4\cos(-x - 3\pi/2) - 2 \Rightarrow x = 4\cos(-y - 3\pi/2) - 2 \Rightarrow x + 2 = 4\cos(-y - 3\pi/2)$

$\Rightarrow \frac{x+2}{4} = \cos(-y - 3\pi/2) \Rightarrow \cos^{-1}\left(\frac{x+2}{4}\right) = -y - 3\pi/2 \Rightarrow \cos^{-1}\left(\frac{x+2}{4}\right) + 3\pi/2 = -y$

$\Rightarrow y = -3\pi/2 - \cos^{-1}\left(\frac{x+2}{4}\right)$

i. cont'd

$P^{-1}(x) = -3\pi/2 - \cos^{-1}\left(\frac{x+2}{4}\right) = -3\pi/2 - \arccos\left(\frac{x+2}{4}\right)$

$R : [-3\pi/2, -\pi/2]$ $D : [-6, 2]$

j. $Q(x) = -\tan(x) + \pi/4$

$D: \frac{(2k+1)}{2}\pi \neq x$ (restricted $(-\pi/2, \pi/2)$)

$R: (-\infty, \infty)$

$y = -\tan x + \pi/4 \Rightarrow x = -\tan y + \pi/4 \Rightarrow (x - \pi/4) = \tan(-y)$

$\tan^{-1}(x - \pi/4) = -y \Rightarrow y = -\tan^{-1}(x - \pi/4)$

$Q^{-1}(x) = -\tan^{-1}(x - \pi/4) = -\arctan(x - \pi/4)$

$D: (-\infty, \infty)$ $R: (-\pi/2, \pi/2)$

k. $R(x) = \arcsin(1/2 x + 1)$ $D: [-4, 0]$ $R: [-\pi/2, \pi/2]$

$y = \arcsin(1/2 x + 1) \Rightarrow x = \arcsin(1/2 y + 1) \Rightarrow \sin x = 1/2 y + 1$

$\Rightarrow \sin x - 1 = \frac{1}{2} y \Rightarrow y = 2\sin x - 2$

$D: (-\infty, \infty)$ $R: [-4, 0]$

(restricted $[-\pi/2, \pi/2]$)

l. $S(x) = -\arccos(x-2) + \pi$

$D: [1, 3]$ $R: [\pi, 2\pi]$

$y = -\arccos(x-2) + \pi \Rightarrow x = -\arccos(y-2) + \pi \Rightarrow x - \pi = -\arccos(y-2)$

$-x + \pi = \arccos(y-2) \Rightarrow \cos(-x+\pi) = y - 2 \Rightarrow y = \cos(-x+\pi) + 2$

$D: (-\infty, \infty)$ $R: [1, 3]$ $S^{-1}(x) = \cos(-x+\pi) + 2$

(restricted $[0+\pi, \pi+\pi] = [\pi, 2\pi]$

m. $T(x) = \arctan(x) + 1$

$D: (-\infty, \infty)$ $R: [-\pi/2 + 1, \pi/2 + 1]$

$y = \arctan x + 1 \Rightarrow x = \arctan y + 1 \Rightarrow x - 1 = \arctan y \Rightarrow \tan(x-1) = y$

$T^{-1}(x) = \tan(x-1)$

$D: \frac{(2k+1)\pi}{2} + 1 \neq x$ (restricted $(-\pi/2 + 1, \pi/2 + 1)$)

$R: (-\infty, \infty)$

n. $U(x) = \operatorname{arccot}(3x-1) + \pi/2$

$D: (-\infty, \infty)$ $R: (\pi/2, 3\pi/2)$

$y = \operatorname{arccot}(3x-1) + \pi/2 \Rightarrow x = \operatorname{arccot}(3y-1) + \pi/2 \Rightarrow x - \pi/2 \Rightarrow \operatorname{arccot}(3y-1)$

$\Rightarrow \cot(x - \pi/2) = 3y - 1 \Rightarrow \cot(x - \pi/2) + 1 = 3y \Rightarrow y = \frac{1}{3}\cot(x - \pi/2) + 1/3$

$U^{-1}(x) = \frac{1}{3}\cot(x - \pi/2) + 1/3$

$D: x \neq k\pi + \pi/2$ (restricted $(0 + \pi/2, \pi + \pi/2) = (\pi/2, 3\pi/2)$)

$R: (-\infty, \infty)$

o. $V(x) = \operatorname{arcsec}(-x-1) + \pi/4$

$D: (-\infty, -2] \cup [0, \infty)$ $R: [\pi/4, 3\pi/4) \cup (3\pi/4, 5\pi/4]$

$y = \operatorname{arcsec}(-x-1) + \pi/4 \Rightarrow x = \operatorname{arcsec}(-y-1) + \pi/4 \Rightarrow$

$x - \pi/4 = \operatorname{arcsec}(-y-1) \Rightarrow \sec(x - \pi/4) = -y - 1 \Rightarrow$

$\sec(x - \pi/4) + 1 = -y \Rightarrow y = -\sec(x - \pi/4) - 1$

$V^{-1}(x) = -\sec(x - \pi/4) - 1$

$D: x \neq \frac{(2k+1)\pi}{2} + \pi/4$ (restricted $[0, \pi/2) \cup (\pi/2, \pi]$ $+\pi/4$ $=$
$[\pi/4, 3\pi/4) \cup (3\pi/4, 5\pi/4]$)

$R: (-\infty, -2] \cup [0, \infty)$

p. $W(x) = \frac{1}{4} \operatorname{arccsc}(\frac{1}{2}x)$ ⑥

D: $(-\infty, -2] \cup [2, \infty)$ R: $[-\pi/8, 0) \cup (0, \pi/8]$

$y = \frac{1}{4}\operatorname{arccsc}(\frac{1}{2}x) \Rightarrow x = \frac{1}{4}\operatorname{arccsc}(\frac{1}{2}y) \Rightarrow 4x = \operatorname{arccsc}(\frac{1}{2}y)$

$\Rightarrow \csc 4x = \frac{1}{2}y \Rightarrow 2\csc 4x = y$

$W^{-1}(x) = 2\csc 4x$

D: $x \neq \frac{k\pi}{4}$ restricted $\left([-\frac{\pi/2}{4}, 0) \cup (0, \frac{\pi/2}{4}]\right.$

R: $(-\infty, -2] \cup [2, \infty)$ $\left. = [-\pi/8, 0) \cup (0, \pi/8]\right)$