

Name KEY
 Math 103, Exam #3, Spring 2012

Instructions: Show all work. If you are using your calculator to solve, you may sketch a graph or indicate keys pressed to show work. Exact values: do not use decimals in your answers unless the problem begins with decimals, or is a word problem. All answers should be fully reduced for full credit.

1. Solve the following equations for all values of the variable that will make the equation true. (7 points each)

a. $(n-7)(n+4) = n+4$

$$(n-7)(n+4) - (n+4) = 0$$

$$(n+4)(n-7-1) = 0$$

$$(n+4)(n-8) = 0$$

$$n = -4, n = 8$$

b. $(b+1)(b-2)(b+3) = 0$

$$b = -1, b = 2, b = -3$$

2. Find three consecutive odd integers such that the product of the first and the third is 96. (10 points)

1st = n
 2nd = $n+2$
 3rd = $n+4$

$$n(n+4) = 96$$

$$n^2 + 4n - 96 = 0$$

$$(n+12)(n-8) = 0$$

$$n = -12, n = 8$$

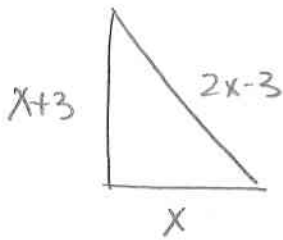
- 1, 96
- 2, 48
- 3, 32
- 4, 24

$$\frac{6, 16}{8, 12}$$

-12	8
-10 or	10
-8	12

Technically, these are even 3 thus the problem has no solution

3. Find the lengths of the sides of a right triangle whose two perpendicular sides are x and $x+3$, and the hypotenuse is $2x-3$. (10 points)



$$x^2 + (x+3)^2 = (2x-3)^2$$

$$x^2 + x^2 + 6x + 9 = 4x^2 - 12x + 9$$

$$0 = 2x^2 - 18x$$

$$2x(x-9)$$

~~$$x=0$$~~

$$x=9$$

$$9^2 + 12^2 = 15^2$$

$$81 + 144 = 225 \checkmark$$

4. Simplify the rational expressions. In each case, find the values of the variable for which the original expression is not defined. (7 points each)

a. $\frac{4x^2 - 20x + 24}{6x^2 - 48x + 90} = \frac{4(x^2 - 5x + 6)}{6(x^2 - 8x + 15)} = \frac{2(x-3)(x-2)}{3(x-3)(x-5)} = \frac{2(x-2)}{3(x-5)}$

$x \neq 3, x \neq 5$ } take from unreduced form

b. $\frac{7a - a^2}{a^3 - 5a^2 - 14a} = \frac{a(7-a)}{a(a^2 - 5a - 14)} = \frac{a(7-a)}{a(a-7)(a+2)} = \frac{-1}{a+2}$

$a \neq 7, a \neq -2, a \neq 0$

5. Find the LCD of the fractions. (5 points each)

a. $\frac{3xy-2y^2-x^2}{x+y}, \frac{x^2-y^2}{x^2-2xy}$
 $x(x-2y)$

LCD: $(x+y)(x-2y)x$

b. $\frac{2}{4x+4}, \frac{8}{3x+3}$
 $4(x+1) \quad 3(x+1)$

LCD: $12(x+1)$

c. $\frac{4}{x^2+2x-15}, \frac{3}{x^2-x-6}$
 $\frac{4}{(x+5)(x-3)}, \frac{3}{(x-3)(x+2)}$

LCD: $(x+5)(x-3)(x+2)$

d. $\frac{4}{x+2}, \frac{-5x-2}{x^2+2x}, \frac{3-x}{x}$
 $x(x+2)$

LCD: $x(x+2)$

6. Simplify. (7 points each)

a. $\frac{3xy-2y^2-x^2}{x+y} \cdot \frac{x^2-y^2}{x^2-2xy} = \frac{-(x^2-3xy+y^2)}{x+y} \cdot \frac{(x+y)(x-y)}{x(x-2y)}$

$\frac{-\cancel{(x-2y)}(x-y)}{\cancel{(x+y)}} \cdot \frac{(x+y)\cancel{(x-y)}}{x\cancel{(x-2y)}}$

$\frac{-(x-y)^2}{x}$

$$b. \frac{p^3 - 8q^3}{p^2 - 4q^2} \div \frac{p^2 + 4pq + 4q^2}{(p+2q)^2}$$

typo corrected

$$\frac{(p-2q)(p+2pq+4q^2)}{(p-2q)(p+2q)} \cdot \frac{(p+2q)^2}{(p+2q)^2} = \boxed{\frac{p+2pq+4q^2}{p+2q}}$$

not corrected

$$\frac{-7p^3}{(p-2q)(p+2q)} \cdot \frac{(p+2q)^2}{(p+2q)^2} = \frac{-7p^3}{(p-2q)(p+2q)}$$

$$c. \frac{3x^2}{x+1} - \frac{x^2+2}{x+1} = \frac{3x^2 - (x^2+2)}{x+1} = \frac{3x^2 - x^2 - 2}{x+1} =$$

$$\frac{2x^2 - 2}{x+1} = \frac{2(x^2-1)}{x+1} = \frac{2(x+1)(x-1)}{x+1} =$$

$$\boxed{2(x-1)}$$

$$d. \frac{3}{4} \frac{2}{4x+4} + \frac{8}{3x+3} \frac{1}{4}$$

$$4(x+1) \quad 3(x+1)$$

$$\frac{6}{12(x+1)} + \frac{32}{12(x+1)} = \frac{38}{12(x+1)} = \frac{19}{6(x+1)}$$

$$e. \frac{4}{x^2+2x-15} + \frac{3}{x^2-x-6}$$

$$(x+5)(x-3) \quad (x-3)(x+2)$$

$$\frac{4(x+2)}{(x+5)(x-3)(x+2)} + \frac{3(x+5)}{(x+5)(x-3)(x+2)} =$$

$$\frac{4x+8 + 3x+15}{(x+5)(x-3)(x+2)} = \boxed{\frac{7x+23}{(x+5)(x-3)(x+2)}}$$

$$f. \frac{4}{x+2} + \frac{-5x-2}{x^2+2x} - \frac{3-x}{x}$$

$$x(x+2)$$

$$\frac{4x}{x(x+2)} + \frac{-5x-2}{x(x+2)} - \frac{(3-x)(x+2)}{x(x+2)}$$

$$\frac{4x - 5x - 2 - (3x + 6 - x^2 - 2x)}{x(x+2)} =$$

$$\frac{4x - 5x - 2 - 3x - 6 + x^2 + 2x}{x(x+2)} = \frac{x^2 - 2x - 8}{x(x+2)} = \frac{(x-4)(x+2)}{x(x+2)}$$

$$\boxed{\frac{x-4}{x}}$$