Instructions: Show all work. Calculators should only be used to check, not perform the work.

1. Find the solution to the second order ODE with constant coefficients. First solve for the homogeneous solution, and then solve the nonhomogeneous case by the method of undetermined coefficients. State the general and particular form for the given set of initial conditions. Show that the homogeneous solutions form a fundamental set.

$$y'' + 2y' + y = 2e^{-t}, y(0) = 2, y'(0) = -3$$

$r^2 + 2r + 1 = 0$

$(r+1)^2 = 0$

$Y_1 = e^{-t}$

$Y_2 = te^{-t}$

$Y(t) = At^2 e^{-t}$

$$Y'(t) = -t^2 e^{-t} + 2te^{-t}$$

$Y''(t) = t^2 e^{-t} - 2te^{-t} - 2te^{-t} + 2e^{-t}$ × A

$2Y'(t) = -2t^2 e^{-t} + 4te^{-t}$ × A

$Y(t) = t^2 e^{-t}$ × A

$t^2 e^{-t}(A - 2A + A) = 0$ $0 = 0$ ✓

$te^{-t}(-4A + 4A) = 0$ $0 = 0$ ✓

$e^{-t}(2A) = 2e^{-t}$ $2A = 2$
$A = 1$

$y(t) = Be^{-t} + Cte^{-t} + t^2 e^{-t}$

$2 = B$

$y'(t) = -Be^{-t} + Ce^{-t} - Cte^{-t} - t^2 e^{-t} + 2te^{-t}$

$-3 = -B + C$

$\begin{array}{r} -3 = -2 + C \\ +2 \quad\quad +2 \\ \hline -1 = C \end{array}$

$\boxed{y(t) = 2e^{-t} - te^{-t} + t^2 e^{-t}}$