Name KEY

Math 285, Quiz #5, Spring 2012

1. Determine the radius and interval of convergence for $\sum_{n=0}^{\infty} \frac{3^n(x-4)^n}{7^n n^2}$.

$$\lim_{n\to\infty} \left| \frac{3^{n+1}(x-4)^{n+1}}{7^{n+1}(n+1)^2} \cdot \frac{7^n n^2}{3^n (x-4)^n} \right| = \lim_{n\to\infty} \left| \frac{3(x-4)}{7} \cdot \frac{n^2}{(n+1)^2} \right| =$$

$$\lim_{n\to\infty} \left| \frac{3(x-4)}{7} \right| \cdot \cancel{\lim_{n\to\infty} \left| \frac{n^2}{(n+1)^2} \right|}^{=1} = \lim_{n\to\infty} \left| \frac{3(x-4)}{7} \right| = \left| \frac{3(x-4)}{7} \right| < 1$$

$$-1 < \frac{3(x-4)}{7} < 1 \Rightarrow -\frac{7}{3} < (x-4) < \frac{7}{3} \Rightarrow 5/3 < x < \frac{19}{3}$$

$+4 \quad +4 \quad +4$

$$\frac{\frac{19}{3} - \frac{5}{3}}{2} = \frac{\frac{14}{3}}{2} = \frac{14}{6} = \frac{7}{3}$$

R: $7/3$

I: $[5/3, 19/3]$

@ $5/3$ $\sum \left(\frac{3}{7}\right)^n \left(-\frac{7}{3}\right)^n \cdot \frac{1}{n^2} = \sum \frac{(-1)}{n^2}$ converges

@ $\frac{19}{3}$ $\sum \left(\frac{7}{3}\right)^n \left(\frac{3}{7}\right)^n \frac{1}{n^2} = \sum \frac{1}{n^2}$ converges

2. Solve the equation $y'' - y = 0$ by using a power series centered around $x_0 = 0$.

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} a_n n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} a_n n(n-1) x^{n-2}$$

$$= \sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n$$

$$\sum a_{n+2}(n+2)(n+1)x^n - \sum a_n x^n = 0$$

$$\sum x^n \left(a_{n+2}(n+2)(n+1) - a_n\right) = 0 \Rightarrow$$

$$a_{n+2}(n+2)(n+1) = a_n \Rightarrow$$

$$\frac{a_n}{(n+2)(n+1)} = a_{n+2}$$

evens $n=0$ odds $n=1$

$n=0$ $\frac{a_0}{2\cdot 1} = a_2$ $n=1$ $\frac{a_1}{3\cdot 2} = \frac{a_1}{3!} = a_3$

$n=2$ $\frac{a_2}{3\cdot 4} = \frac{a_0}{4!} = a_4$ $n=3$ $\frac{a_3}{5\cdot 4} = \frac{a_1}{5!} = a_5$

$n=4$ $\frac{a_4}{5\cdot 6} = \frac{a_0}{6!} = a_6$

⋮

$$\sum \frac{a_0}{(2k)!} x^{2k} + \sum \frac{a_1}{(2k+1)!} x^{2k+1}$$

evens odds

$a_0 \sinh x + a_1 \cosh x$