Name KEY

Math 285, Quiz #6, Spring 2012

Instructions: Show all work. You may use a calculator to check your work, but to receive full credit for any calculus, you must show the steps.

1. Solve the boundary value problem $y'' + 4y' - 5y = 0, y(0) = 0, y(1) = 0$. Determine if the solution is unique, represents an infinite set of solutions, or does not exist.

$r^2 + 4r - 5 = 0$

$(r+5)(r-1) = 0$

$r = -5, 1$

$y = 0$ trivial solution only

$y = Ae^{-5x} + Be^{x}$

$0 = A + B \qquad A = -B$

$0 = Ae^{-5} + Be = Ae^{-5} - Ae = A(e^{-5} - e) \Rightarrow A = 0$

$e^{-5} - e \neq 0$

$\Rightarrow B = 0$

2. Find the eigenvalues and corresponding eigenfunctions for the system $y'' + 2\lambda y' + \lambda^2 y = 0, y(0) = 0, y(1) = L$ where L is some real number greater than zero. Describe the set of eigenvalues. Is the set finite, or infinite? If it's infinite, does it map onto the set of natural numbers, or real numbers?

$\lambda > 0, \lambda < 0$

$r^2 + 2\lambda r + \lambda^2 = 0$

$(r+\lambda)^2 = 0$

$-\lambda = r$

$y = Ae^{-\lambda x} + Bxe^{-\lambda x}$

$0 = A + 0 \Rightarrow A = 0$

$L = B(1)e^{-\lambda}$

$B = Le^{\lambda}$

$y = Le^{\lambda}xe^{-\lambda x} = Lxe^{-\lambda x + \lambda}$

$= Lxe^{-\lambda(x-1)}$

λ any real # $\neq 0$

$\lambda = 0$

$y'' = 0$

$y = Ax + B$

$0 = A(0) + B \Rightarrow B = 0$

$L = A$

$y = Lx$

for $\lambda = 0$

Eigenvalues

Set is infinite, maps onto real #'s