Math 1116, Exam #1, Spring 2013 Name KEY

Instructions: Show all work for full credit. Circle your final answers. Problems where you are asked to define a term or explain a concept, use complete sentences.

1. Consider the preference table below and answer the questions about it which follow.

	10	8	4	4	3
1st choice	B	A	B	C	C
2nd choice	A	B	C	B	A
3rd choice	C	C	A	A	B

a. Calculate the winner of the election by the plurality method and then rank the remaining candidates.

A = 8 B = 14 C = 7

B wins
A second
C third

b. How many votes are needed to win the election if a majority is required?

29 total votes / 2 = 14.5 → 15 needed

c. Calculate the winner by the plurality with elimination method.

A = 8 B = 14 ~~C = 7~~

A = 17 B = 18

B wins

d. Calculate the winner by the Borda Count method.

A = 10×2 + 8×3 + 4×1 + 4×1 + 3×2 = 58

B = 10×3 + 8×2 + 4×3 + 4×2 + 3×1 = 69

C = 10×1 + 8×1 + 4×2 + 4×3 + 3×3 = 47

B wins

e. Calculate the winner by the method of pairwise comparisons.

A-B : B=18 A=11 B gets 1 A=1

A-C : A=18 C=11 A gets 1. B=2

B-C : B=22 C=7 B gets 1 C=0

B wins

f. Is there a Condorcet criterion violation? What about a majority criterion violation?

No Since B wins by all methods, and there was no initial majority winner

g. What would be required to create a monotonicity criterion violation?

We would need 2 votes to compare and in the second vote more votes would have needed to go to B, but Cause B to lose

h. What is the Independence of Independent Alternatives criterion? Explain how it works.

This criterion says if a loser drops out, the winner of the election will change. For instance, if C dropped out but all votes for C went to A, then A would win (which won't happen here).

i. What is Arrow's Impossibility Theorem? What does it say?

it says that no voting method can satisfy all fairness criterion all the time.

2. Consider the weighted voting system [q: 11, 10, 9, 4]. Use this system to answer the following questions.

   a. What is the minimum value the quota can take? What is the maximum?

min q = 18

max q = 34

34/2 = 17

+ 1

18

   b. What is the value of the quota if a minimum of 60% of the votes are needed to pass? Use this value in all the remaining questions.

34 x .6 = 20.4 → 21

   c. List all the winning coalitions and indicate which players are critical in each coalition.

[21 : 11, 10, 9, 4]

{P1, P2, P3, P4}

{P1, P2, P3} {P1, P3, P4}

{P1, P2, P4}

{P2, P3, P4}

{P1, P2}

   d. Use the information in part c to calculate the Banzhaf power distribution for this system.

Critical players = 12

P1 : $\frac{4}{12}$ = 33%

P2 : $\frac{4}{12}$ = 33%

P3 = $\frac{2}{12}$ = 17%

P4 = $\frac{2}{12}$ = 17%

no dummies
no veto power

e. List all the sequential coalitions for this system and indicate the pivotal players in each coalition.

[21, 11, 10, 9, 4]

<P1, P2, P3, P4>
<P1, P2, P4, P3>
<P1, P3, P2, P4>
<P1, P3, P4, P2>
<P1, P4, P2, P3>
<P1, P4, P3, P2>
<P2, P1, P3, P4>
<P2, P1, P4, P3>
<P2, P3, P1, P4>
<P2, P3, P4, P1>
<P2, P4, P1, P3>
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<P3, P4, P2, P1>
<P4, P1, P2, P3>
<P4, P1, P3, P2>
<P4, P2, P1, P3>
<P4, P2, P3, P1>
<P4, P3, P1, P2>
<P4, P3, P2, P1>

f. Use the information in part e to calculate the Shapley-Shubik power distribution for this system.

$P1 = \frac{8}{24} = 33\%$

$P2 = \frac{8}{24} = 33\%$

$P3 = \frac{4}{24} = 17\%$

$P4 = \frac{4}{24} = 17\%$

g. Compare the results from parts d and f to the percentage each player has relative to the other by weight. How fair to do you think this weighted voting system is as is?

by weight

$P1 = \frac{11}{34} = 32\%$

$P2 = \frac{10}{34} = 29\%$

$P3 = 26\%$

$P4 = 12\%$

pretty close for P1, P2, but P3 has much less power & P4 much more than weighting suggests.