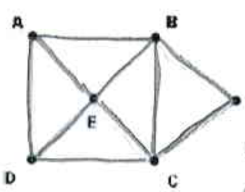
Math 1116, Exam #2, Spring 2013 Name KEY

Instructions: Show all work. Complete all parts of each question, and answer as fully as possible. Use correct notation.

1. Determine whether the graphs below have each of the following characteristics:
   a. Is the graph connected?
   b. Does the graph have an Euler circuit? If not, does it have an Euler path?
   c. Does the graph have a Hamilton circuit? If not, does it have a Hamilton path?
   d. Is the graph a tree?
   e. Is the graph complete?
   f. How many bridges does the graph contain?

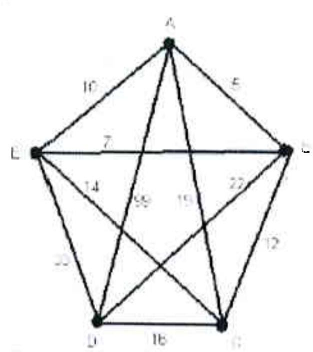
Be sure to check each graph below for all 6 characteristics. Each characteristic of each graph is 1 point.

i.



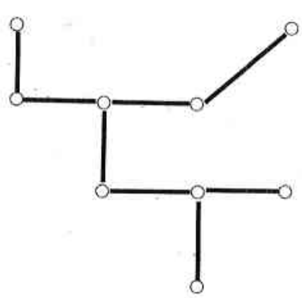
a. yes
b. path
c. circuit
d. no
e. no
f. 0

ii.



a. yes
b. circuit
c. circuit
d. no
e. yes
f. 0

iii.



a. yes
b. none
c. none
d. yes
e. no
f. 8

iv.

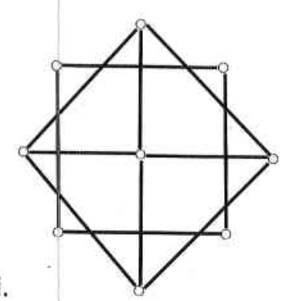
a. yes
b. path
c. circuit
d. no
e. no
f. 0

v.

a. no
b. none
c. none
d. no
e. no
f. 3 or none

Since graph is already disconnected

vi.



a. no
b. none
c. none
d. no
e. no
f. 0 / graph is already disconnected

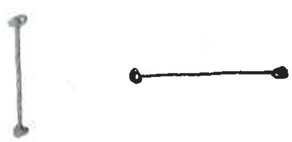
2. For each of the conditions below, draw a graph with the stated properties. (5 points each)
   a. A connected graph with 5 vertices.

answers will vary

   b. A connected graph where one vertex has degree 6.

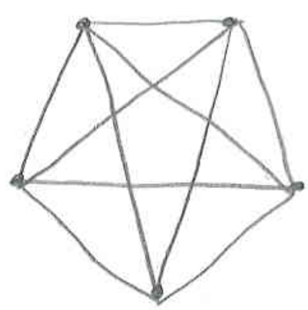
answers will vary

   c. A disconnected graph.



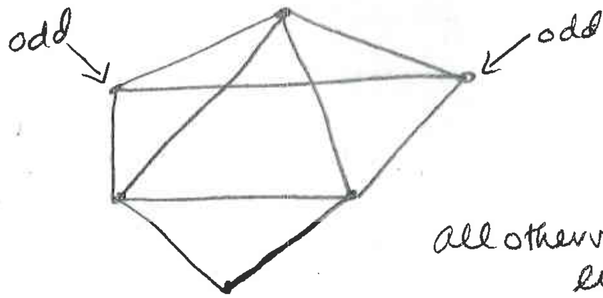
answers will vary

   d. A complete graph.



answers will vary

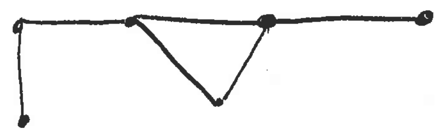
e. A graph with an Euler path, but not an Euler circuit.



answers will vary

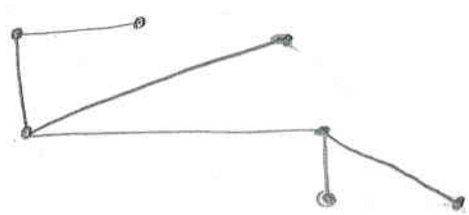
all other vertices even

f. A graph that contains exactly three bridges and one circuit.



answers will vary

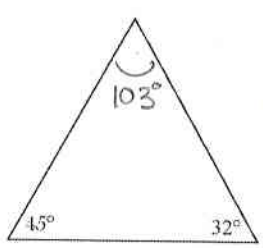
g. A tree with 7 vertices.



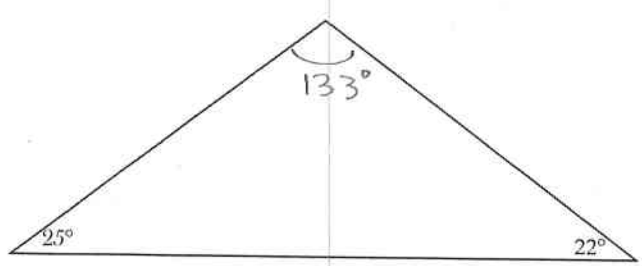
answers will vary

3. Determine if either or both (or neither) of the networks graphed below require the construction of a Steiner point to find a minimal length network. (5 points each)

a.



b.



180- 45 - 32 = 103

needs a Steiner point

180-25-22 = 133

no Steiner point

4. Determine if each statement is True or False. (3 points each)

a. (T) F The redundancy of a graph is the number of edges that can be removed and still have a connected graph.

b. T (F) A Steiner point is a native junction.

c. T (F) AB is not the same edge as BA.

d. (T) F A loop is an edge that connects a vertex to itself.

e. (T) F An Euler path may contain no odd vertices.

f. T (F) An Euler circuit may repeat edges but not vertices.

g. (T) F A semi-Eulerized graph is obtained by duplicating edges until all but two vertices are of even degree.

h. T (F) A complete graph has N vertices and each vertex is of degree N.

i. T (F) It's possible to construct a graph with one vertex of odd degree.

j. (T) F An efficient procedure is something that takes comparatively little work to find the result.

k. T (F) An Euler path must start at a vertex of even degree.

l. T (F) An approximate procedure finds the best possible result.

m. T (F) A tree always contains at least one circuit.

n. T (F) Kruskal's algorithm is inefficient but optimal.

o. T (F) The minimal spanning tree is the tree with the fewest edges.

5. Define the following terms: (5 points each)

a. The degree of a vertex

the number of edges emerging from that vertex

b. Fluery's Algorithm

- Choose starting point.
- choose vertices and "delete" edges from remaining graph as you go
- choose no edge that is a bridge to the unfinished portion of the graph until you have used all edges.

c. Hamilton circuit

a circuit That visits all vertices of a graph exactly once until it returns to The starting vertex

d. Optimal

the best possible Solution
in our problems: the graph w/ The least possible weight

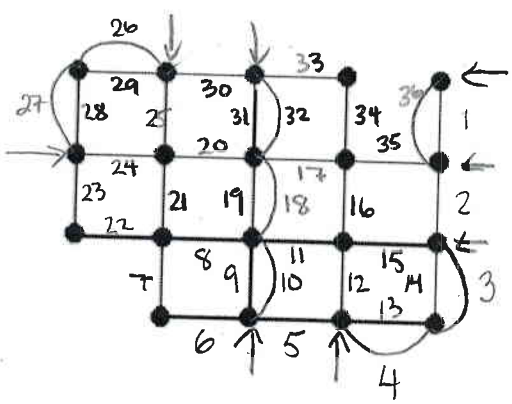
6. For a complete graph of 8 vertices, calculate the number of unique Hamilton circuits that would be possible. (5 points)

$$\frac{(8-1)!}{2} = \frac{7!}{2} = \frac{5040}{2} = 2520$$

7. In an ideal situation, a graph with 8 odd vertices can be semi-Eulerized with no fewer than how many edges? (5 points)

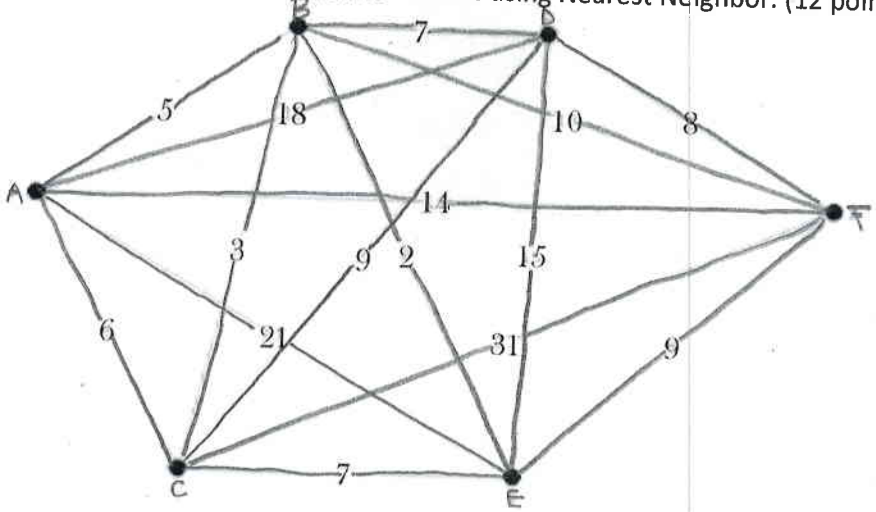
$$\frac{8-2}{2} = \frac{6}{2} = 3$$

8. Consider the graph below. Determine if the graph has an Euler circuit or path. If it does not, explain why not. Then Eulerize the graph (if needed). Find an Euler circuit. Label the edges as you use them. (You may label the vertices if you wish, but this is not necessary.) (12 points)



no Euler path or circuit now
8 odd vertices

9. For the graph below, find a Hamilton circuit using Nearest Neighbor. (12 points)



Choose one Starting pt.

ABCEFDA
5+3+7+9+8+18 = 50

CBEFDAC
3+2+9+8+18+6 = 46

EBCAFDE
2+3+6+14+8+15 = 48

BECAFDB
2+7+6+14+8+7 = 44

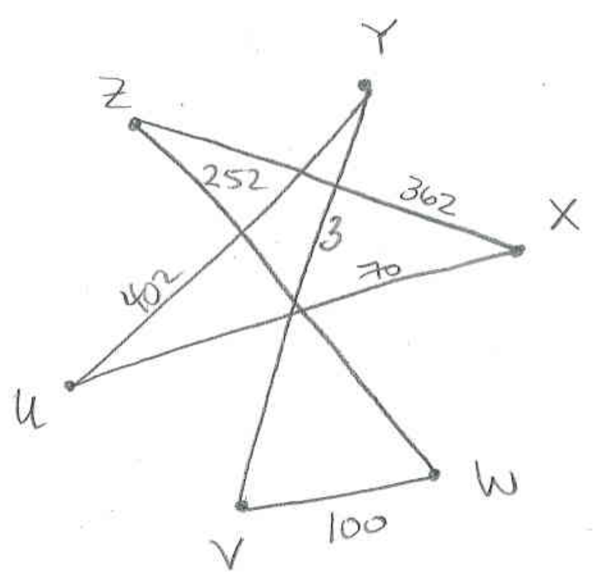
DBECAFD
7+2+7+6+14+8 = 44

FDBECAF
8+7+2+7+6+14 = 44

10. Use Cheapest link to find a Hamilton circuit on the data in the table below. (12 points)

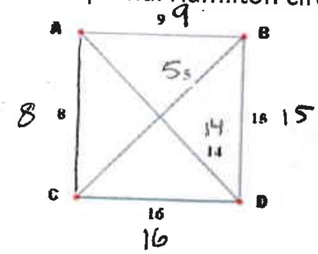
	Zatar	Yolanda	Xerxes	Wanda	Veronica	Ulysses
Zatar	-	339	362	252	599	546
Yolanda		-	634	112	3	402
Xerxes			-	618	798	70
Wanda				-	100	995
Veronica					-	189
Ulysses						-

YVWZXUY
3+100+252+362+70+402



= 1189

11. Use Brute Force to find the optimal Hamilton circuit on the graph below. (12 points)



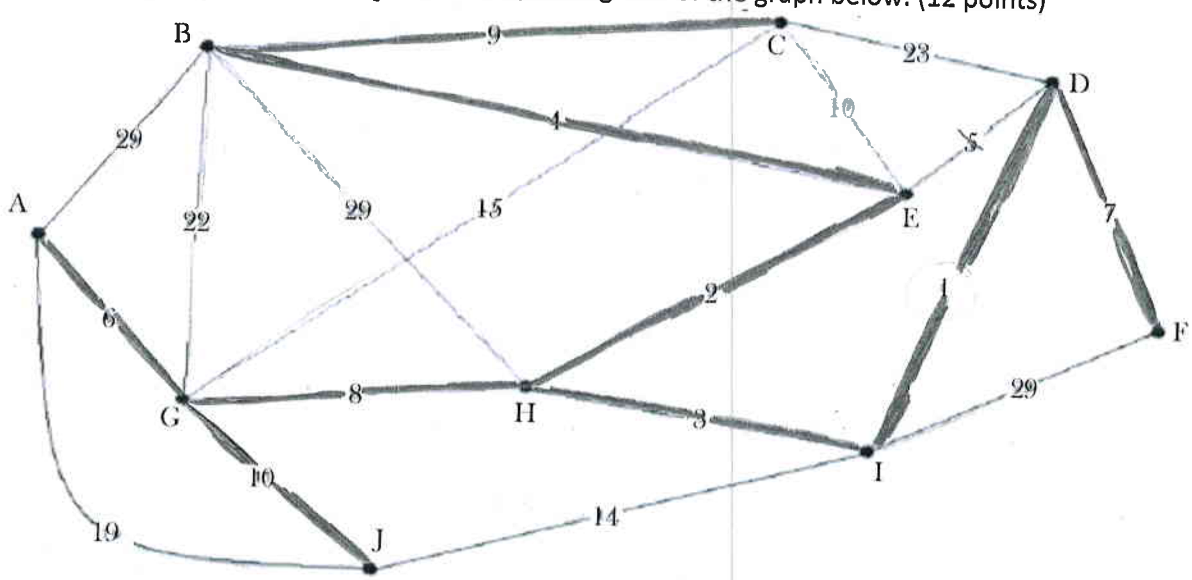
$\frac{(4-1)!}{2} = 3$

ABCDA = 9+5+16+14 = 44

ABDCA = 9+15+16+8 = 48

ACBDA = 8+5+15+14 = 42 optimal

12. Use Kruskal's algorithm to find the minimal spanning tree of the graph below. (12 points)



1+2+3+4+6+7+8+9+10 = 50