

Math 1116 Homework #1 Key

i. 460 ii. $230+1=231$ iii. $115+1=116$

iv. $A=150, B=50, C=120, D=140$ A wins

v. 1st = 4 pts 2nd = 3 pts 3rd = 2 pts 4th = 1 pts

$$A = 120 \times 1 + 50 \times 2 + 40 \times 3 + 90 \times 4 + 60 \times 4 + 100 \times 2 = 1140 \text{ pts}$$

$$B = 120 \times 2 + 50 \times 4 + 40 \times 2 + 90 \times 2 + 60 \times 1 + 100 \times 2 = 860 \text{ pts}$$

$$C = 120 \times 4 + 50 \times 3 + 40 \times 1 + 90 \times 3 + 60 \times 2 + 100 \times 1 = 1160 \text{ pts}$$

$$D = 120 \times 3 + 50 \times 1 + 40 \times 4 + 90 \times 1 + 60 \times 3 + 100 \times 4 = 1240 \text{ pts}$$

D wins

vi. A-B: $B=120, B=50, A=40, A=90, A=60, B=100, 190 \text{ vs. } 270 = B$

A-C: $C=120, C=50, A=40, A=90, A=60, A=100, 290 \text{ vs. } 170 = A$

A-D: $D=120, A=50, D=40, A=90, A=60, D=100, 200 \text{ vs. } 260 = D$

B-C: $C=120, B=50, B=40, C=90, C=60, B=100, 190 \text{ vs. } 270 = C$

B-D: $D=120, B=50, D=40, B=90, D=60, D=100, 140 \text{ vs. } 320 = D$

C-D: $C=120, C=50, D=40, C=90, D=60, D=100, 260 \text{ vs. } 200 = C$

C & D both win in a tie

A = 1 pt. B = 1 pt. C = 2 pt. D = 2 pt.

vii. A does not have a majority, B received the fewest votes, so eliminate B. If B is eliminated C is 2nd on all his ballots, recount as 1st.

$$A = 150 \quad \del B \quad C = 170 \quad D = 140$$

Still no one has 231 votes. Next lowest is D, so eliminate D.

40 votes for D go to A

100 votes for D go to B but he's out already, so A is next

140 for A

$$A = 290 \quad \del B \quad C = 170 \quad \del D \quad \text{A wins}$$

viii. There are 2 Condorcet candidates (vi) neither of which match the plurality winner so that's a violation

no one had a majority, so no majority criterion violation

no monotonicity violation since we need to compare successive votes w/ same candidates

no IIA violation since no one dropped out of the race

16. i. 44 ii. 22+1=23 iii. 1+1=12

iv. A=18, B=11, C=9 D=6 A wins

v. A = 18x4 + 11x1 + 9x1 + 6x1 = 98

B = 18x2 + 11x4 + 9x3 + 6x2 = 125 B wins

C = 18x1 + 11x3 + 9x4 + 6x3 = 105

D = 18x3 + 11x2 + 9x2 + 6x4 = 118

vi. A-B: A=18 B=11 B=9 B=6 18 vs. 26 B

A-C: A=18 C=11 C=9 C=6 18 vs 26 C

A-D: A=18 D=11 D=9 D=6 18 vs. 26 D

B-C: B=18 B=11 C=9 C=6 29 vs 15 B

B-D: B=18 B=11 B=9 D=6 20 vs 24 D

C-D: D=18 C=11 C=9 D=6 20 vs 24 D

A=0, B=2, C=1, D=3 D wins

vii. A=18 B=11 C=9 ~~D=6~~ → C

A=18 ~~B=11~~ → C C=15

A=18 C=26 C wins

viii. Condorcet candidate is D. all methods violate except Copeland's
no majority, so this is okay in all cases.
no monotonicity since no other set of ballots to compare w/
no IIA since no one dropped out of race

2. if a voter supports an unpopular candidate who cannot win, a voter
might choose to vote for their second place candidate instead of
their true first choice in hopes of boosting that candidate's chances
of winning over the less preferred third choice. In the US with
2 strong parties, a third party candidate's voters are heavily pressured
to vote for which of the two party candidates most closely matches their
views.

3. Your answers will depend on what you want to achieve. I will give examples of good answers. There are no "correct" answers, only well-reasoned ones. (3)

a. Use Borda Count, because it tends to reduce the likelihood of polarizing candidates winning and consensus candidates tend to win.

b. Instant-run off voting because the plurality method alone reduces the party system to just 2 parties leaving a lot of people feeling unrepresented and change is difficult. Instant run-off gives third parties a chance, as well as independents, but if they don't achieve enough votes, can still be eliminated without the expense of a separate run-off vote. (Of course, more parties also means more elections could get thrown into the House which could be bad.)

c. Since one doesn't want home-coming queen to be polarizing, either Borda Count or Copeland's method.

d. Borda Count or instant run-off depending on how important the majority criterion is to you.

e. Copeland's method, since Borda Count is typically used here, if something is wrong with it, maybe this will give different results, but Borda Count may be the best here since the irrelevance of independent alternatives criterion may matter a lot in this circumstance.