Math 2255, Quiz #6, Spring 2013 Name KEY

Instructions: Show all work. Solve the problems as completely as possible with the given information. Use exact answers unless specifically asked to round.

1. Solve the second order differential equations.
   a. $y'' + 8y' + 16y = 0, y(0) = 1, y'(0) = 4$

$r^2 + 8r + 16 = 0$

$(r+4)^2 = 0 \quad r = -4$

$y_c(t) = c_1 e^{-4t} + c_2 t e^{-4t}$

$1 = c_1$

$y_c'(t) = -4e^{-4t} + c_2 e^{-4t} - 4c_2 t e^{-4t}$

$4 = -4 + c_2 \Rightarrow c_2 = 8$

$$\boxed{y_c(t) = e^{-4t} + 8t e^{-4t}}$$

   b. $t^2y'' - 6ty' + 12y = 0, y(1) = 2, y(2) = 6$

$n^2 - n - 6n + 12 = 0$

$n^2 - 7n + 12 = 0$

$(n-3)(n-4) = 0 \quad n = 3, 4$

$y_c(t) = c_1 t^3 + c_2 t^4$

$2 = c_1 + c_2$

$6 = 8c_1 + 16c_4$

$\Rightarrow -16 = -8c_1 - 8c_2$

$6 = 8c_1 + 16c_2$

$\frac{-10}{8} = \frac{8c_2}{8}$

$c_2 = \frac{-5}{4}$

$c_1 = 2 - (\frac{-5}{4}) = \frac{8}{4} + \frac{5}{4} = \frac{13}{4}$

$$\boxed{y_c(t) = \frac{13}{4}t^3 - \frac{5}{4}t^4}$$

2. Use reduction of order to find the second solution to the differential equation $(1 - 2x - x^2)y'' + \underbrace{2(1 + x)}_{2+2x}y' - 2y = 0$ given that $y_1 = x + 1$.

$y = v(x)(x+1)$

$y' = v'(x)(x+1) + v(x)(1)$

$y'' = v''(x)(x+1) + v'(x)(2)$

$v''(x)(x+1)(1-2x-x^2) + 2v'(x) - \cancel{4xv'(x)} - \cancel{2x^2v'(x)} + \cancel{2xv'(x)} + 2v'(x) + \cancel{2x^2v'(x)}$
$+ \cancel{2xv'(x)} + \cancel{2v(x)} + \cancel{2xv(x)} - \cancel{2v(x)} - \cancel{2xv(x)}$

let $u = v'(x) \quad v''(x) = u'$

$v''(x)(x+1)(1-2x-x^2) + 4v'(x) = 0$

$$\int \frac{u'}{u} = \frac{-4}{(x+1)(1-2x-x^2)} = \int \frac{4}{(x+1)(x^2+2x-1)} dx$$

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$$\frac{A}{x+1} + \frac{Bx+C}{x^2+2x-1} = A(x^2+2x-1) + (Bx+C)(x+1) = 4$$

$x=-1$ $\quad A(\cancel{1}-2-\cancel{1}) = 4 \Rightarrow -2A = 4 \quad A = -2$

$x=0$ $\quad -2(-1) + (C)(1) = 4 \Rightarrow C = 2$

$x=1$ $\quad -2(\cancel{1}+2-\cancel{1}) + (B+2)(2) = 4 \Rightarrow -4 + 2(B+2) = 4$

$\Rightarrow 2(B+2) = 8 \Rightarrow B+2 = 4 \Rightarrow B = 2$

$$\int \frac{du}{u} = \int \frac{-2}{x+1} + \frac{2x+2}{x^2+2x-1}\, dx \Rightarrow$$

$$\ln u = -2\ln|x+1| + \ln|x^2+2x-1| \Rightarrow u = \frac{x^2+2x-1}{(x+1)^2} = \frac{x^2+2x-1}{x^2+2x+1}$$

$$\begin{array}{r} 1 \\ x^2+2x+1 \overline{\big) \, x^2+2x-1} \\ -\,x^2 \mp 2x \mp 1 \\ \hline -2 \end{array} = 1 - \frac{2}{(x+1)^2}$$

$$\int v' = \int 1 - \frac{2}{(x+1)^2}\, dx = x + \frac{2}{x+1} + C$$

$y_1 = x+1 \qquad y_2 = \left(x + \frac{2}{x+1}\right)(x+1) = x(x+1) + \frac{2\cancel{(x+1)}}{\cancel{x+1}} = x^2+x+2$

$$y(t) = c_1(x+1) + c_2(x^2+x+2)$$