Math 2255, Quiz #8, Spring 2013 Name KEY

1. Solve the homogeneous higher order equations.

a. $t^3y''' - 3ty' + y = 0$

$n(n-1)(n-2) - 3n + 1 = 0$

$n(n^2-3n+2) - 3n + 1 = n^3 - 3n^2 + 2n - 3n + 1 = n^3 - 3n^2 - n + 1 = 0$

3 real roots

$n = -.6751309$

$n = .46081113$

$n = 3.2143197$

$y(t) \approx c_1 t^{-.675} + c_2 t^{.461} + c_3 t^{3.214}$

b. $2y^{IV} + 13y''' - 5y'' + 13y' - 7y = 0$

$2r^4 + 13r^3 - 5r^2 + 13r - 7 = 0$

$r^4 + \frac{13}{2}r^3 - \frac{5}{2}r^2 + \frac{13}{2}r - \frac{7}{2}$

$r = -7 \quad r = \frac{1}{2}$

1/2	1	13/2	-5/2	13/2	-7/2
		1/2	7/2	1/2	7/2
-7	1	7	1	7	0
		-7	0	-7	
	1	0	1	0	

$(r+7)(2r-1)(r^2+1)$

$r = \pm i$

$y(t) = c_1 e^{-7} + c_2 e^{\frac{1}{2}t} +$

$c_3 \cos t + c_4 \sin t$

2. The equation $y''' + y'' - 6y' = 0$ has the solutions $y_1 = 1, y_2 = e^{2t}, y_3 = e^{-3t}$. Find the value of the Wronskian for this set of solutions. Do they form a fundamental set?

$$\begin{vmatrix} 1 & e^{2t} & e^{-3t} \\ 0 & 2e^{2t} & 3e^{-3t} \\ 0 & 4e^{2t} & 9e^{-3t} \end{vmatrix} = 1\left(2e^{2t} \cdot 9e^{-3t} + 4e^{2t} 3e^{-3t}\right) + 0$$

$$18e^{-t} + 12e^{-t} = 30e^{-t}$$

yes, fundamental set