Math 2568, Quiz #13, Spring 2013 Name KEY

Instructions: Show all work. Answers must be justified for full credit. Use exact answers unless otherwise specified.

1. Suppose A is a 4x4 matrix with eigenvalues 0, 1, 2, with the eigenvalue 1 repeated. What conditions would have to be satisfied to ensure that the matrix was diagonalizable and what would that D matrix look like?

the eigenvalue 1 would need to produce a 2 dimensional eigenspace

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

2. Find a similarity transformation for the matrix $A = \begin{bmatrix} 2 & 2 \\ -13 & -8 \end{bmatrix}$. State the similarity transformation matrix P and the resulting matrix.

$$\begin{bmatrix} 2-\lambda & 2 \\ -13 & -8-\lambda \end{bmatrix} \Rightarrow (2-\lambda)(-8-\lambda)+26 = -16-2\lambda+8\lambda+\lambda^2+26$$

$$\lambda^2+6\lambda+10=0$$

$$P = \begin{bmatrix} -5 & -1 \\ 13 & 0 \end{bmatrix}$$

$$\lambda = \frac{-6 \pm \sqrt{36-40}}{2} = \frac{-6 \pm 2i}{2} = -3 \pm i$$

$$C = \begin{bmatrix} -3 & -1 \\ 1 & -3 \end{bmatrix}$$

$-3+i$

$$\begin{bmatrix} 2-(-3+i) & 2 \\ -13 & -8-(-3+i) \end{bmatrix} = \begin{bmatrix} 5-i & 2 \\ -13 & -5-i \end{bmatrix}$$

$-13x_1 - (5+i)x_2 = 0$

$x_1 = \frac{5+i}{-13} x_2$

$x_2 = x_2$

$$\vec{v}_1 = \begin{bmatrix} -5 \\ 13 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} i$$

$$\vec{v}_2 = \begin{bmatrix} -5 \\ 13 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} i$$

$-3-i$

3. For the vectors $\vec{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} -6 \\ 9 \end{bmatrix}$, find the following:
   a. $\|\vec{u}\| = \sqrt{4+9} = \sqrt{13}$

   b. $\vec{u} \cdot \vec{v} = -12 + 27 = 15$

   c. Are $\vec{u}$ and $\vec{v}$ orthogonal? no