Math 2568, Quiz #3, Spring 2013 Name KEY

Instructions: Show all work. Be sure to answer all parts of each question. Use exact answers unless specifically asked to round.

1. There are three "tests" for determining if a function T is a linear transformation:
   i) $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$
   ii) $T(c\vec{u}) = cT(\vec{u})$
   iii) $T(\vec{0}) = \vec{0}$

   a. For the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, show, using generic vectors, that this is a linear transformation.

let $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ $\vec{u}+\vec{v} = \begin{bmatrix} u_1+v_1 \\ u_2+v_2 \end{bmatrix}$ $c\vec{u} = \begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix}$

① $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} u_1+v_1 \\ u_2+v_2 \end{bmatrix} = \begin{bmatrix} u_1+v_1-u_2-v_2 \\ 2u_1+2v_1+3u_2+3v_2 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} u_1-u_2 \\ 2u_1+3u_2 \end{bmatrix} + \begin{bmatrix} v_1-v_2 \\ 2v_1+3v_2 \end{bmatrix} = \begin{bmatrix} u_1+v_1-u_2-v_2 \\ 2u_1+2v_1+3u_2+3v_2 \end{bmatrix}$ ✓

② $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix} = \begin{bmatrix} cu_1 - cu_2 \\ 2cu_1+3cu_2 \end{bmatrix}$ $c\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = c\begin{bmatrix} u_1-u_2 \\ 2u_1+3u_2 \end{bmatrix} = \begin{bmatrix} cu_1-cu_2 \\ 2cu_1+3cu_2 \end{bmatrix}$ ✓

③ $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0-0 \\ 0+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ✓

   b. Show that the derivative operator $\frac{d}{dx}$ is a linear operator, using generic functions and properties of derivatives you learned in Calc I.

① $\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$ ✓

② $\frac{d}{dx}[k\,f(x)] = k\frac{d}{dx}[f(x)]$ ✓

③ $\frac{d}{dx}[0] = 0$ ✓