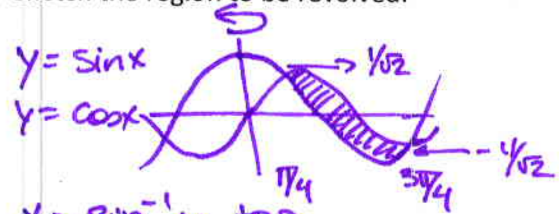
Math 1152, Quiz #15, Spring 2014 Name KEY

Instructions: Show all work. Use *exact* answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. Find the volume of the solid of revolution formed by revolving the area bounded by $y = \sin x$ and $y = \cos x$ around the y-axis using the washer method. Sketch the region to be revolved.

See next page

do not integrate



$x = \sin^{-1} y$ top

$x = \cos^{-1} y$ bottom

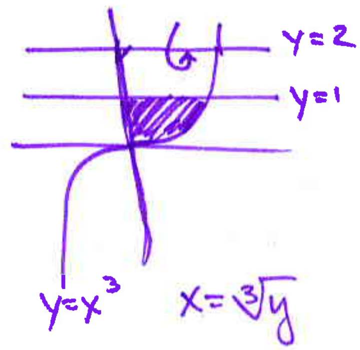
$x = 0$

2. Find the volume of the region bounded by $y = x^3$ and $y = 1$ revolved around the line $y = 2$. Sketch the region to be revolved. Use the shell method.

$$V = 2\pi \int_0^1 (2-y)(\sqrt[3]{y})\, dy$$

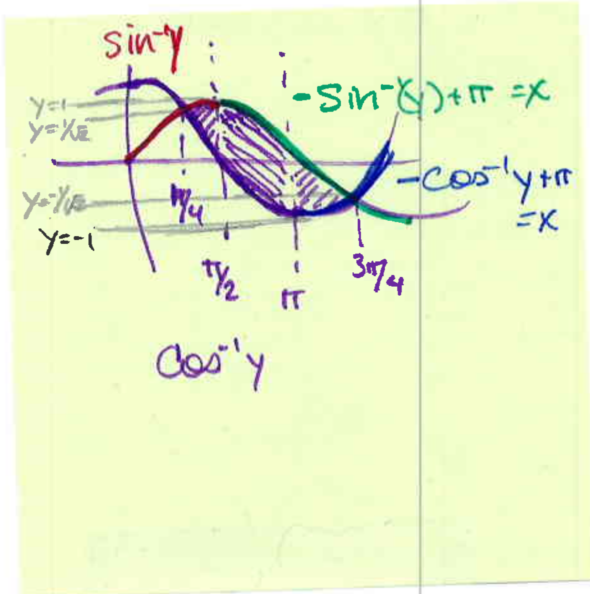
$$2\pi \int_0^1 2y^{1/3} - y^{4/3}\, dy =$$

$$2\pi \left[\frac{3}{\cancel{4}_2}\cancel{2} y^{4/3} - \frac{3}{7} y^{7/3} \Big|_0^1 \right] = 2\pi \left[\frac{3}{2} - \frac{3}{7} \right] = \frac{15\pi}{7}$$

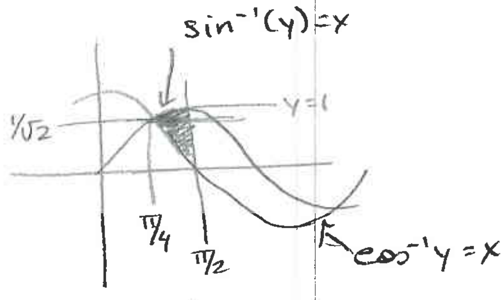


3. Explain when you would use the washer or the shell methods if the problem doesn't tell you which to use?

ideally, when revolving around x-axis if functions are in x-variables (or around y if f(y)) use washer. if revolving around x & f(y) or revolving around y-axis w/ f(x) use shell method



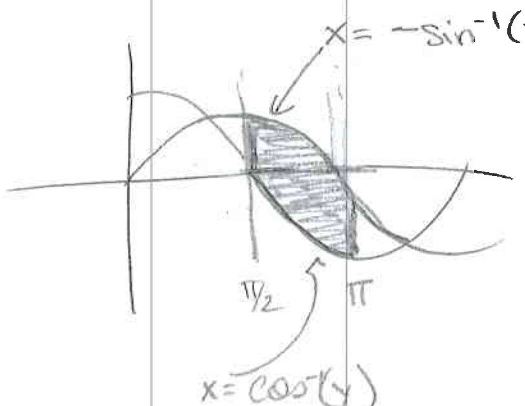
Region #1



$$V=\pi\int_{1/\sqrt{2}}^{1}\left(\tfrac{\pi}{2}\right)^2-\left(\sin^{-1}y\right)^2\,dy\;+$$

$$\pi\int_{0}^{1/\sqrt{2}}\left(\tfrac{\pi}{2}\right)^2-\left(\cos^{-1}y\right)^2\,dy$$

Region #2

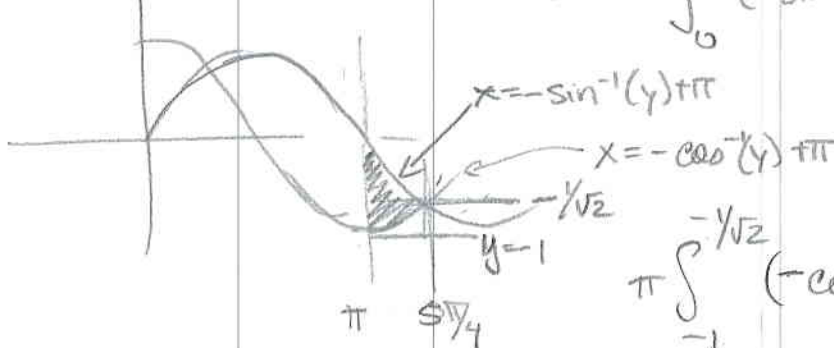


$$V=\pi\int_{0}^{1}\left(-\sin^{-1}(y)+\pi\right)^2-\left(\tfrac{\pi}{2}\right)^2dy$$

$$+\;\pi\int_{-1}^{0}\left(-\sin^{-1}(y)+\pi\right)^2-\left(\cos^{-1}y\right)dy$$

Region #3

$$V=\pi\int_{0}^{1/\sqrt{2}}\left(-\sin^{-1}(y)+\pi\right)^2-(\pi)^2\,dy\;+$$



$$\pi\int_{-1}^{-1/\sqrt{2}}\left(-\cos^{-1}(y)+\pi\right)^2-(\pi)^2\,dy$$

add up all 6 pieces

So much easier to use shell method!