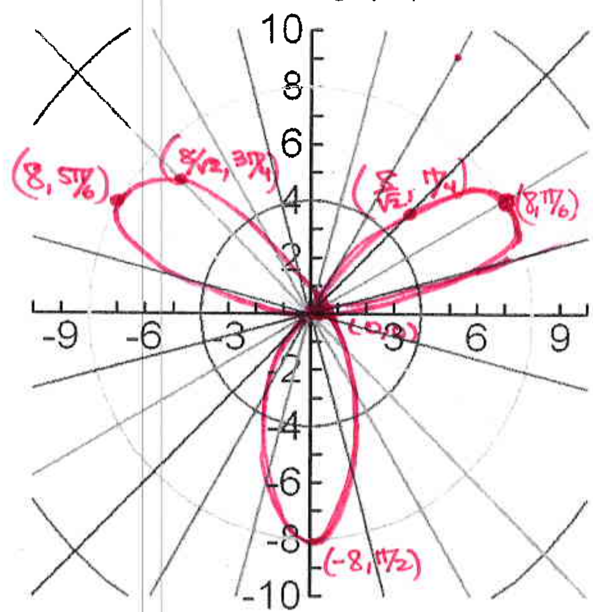
Math 1152, Quiz #8, Spring 2014 Name KEY

Instructions: Show all work. Use *exact* answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. Sketch the graph of $r = 8\sin 3\theta$. Plot and label a minimum of 8 points on the graph (with exact values). Solve for any poles of the graph.

θ	r
0	0
$\pi/6$	8
$\pi/4$	$8/\sqrt{2}$
$\pi/3$	0
$\pi/2$	-8
$2\pi/3$	0
$3\pi/4$	$8/\sqrt{2}$
$5\pi/6$	8
π	0



2. Find the area swept out by one petal of the graph in Problem #1.

$$A = \frac{1}{2}\int_0^{\pi/3} (8\sin 3\theta)^2 \, d\theta = 32\int_0^{\pi/3} \sin^2 3\theta \, d\theta = 16\int_0^{\pi/3} 1 - \cos 6\theta \, d\theta$$

$$= 16\left[\theta - \frac{1}{6}\sin 6\theta\right]_0^{\pi/3} = \frac{16\pi}{3}$$

3. Using the formula $\frac{dy}{dx} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$, find the slope of the tangent line to the equation in Problem #1, at the point $\theta = \frac{\pi}{4}$.

$\sin \pi/4 = 1/\sqrt{2} = \sqrt{2}/2$

$\sin 3\pi/4 = +1/\sqrt{2} = +\sqrt{2}/2$

$\cos \pi/4 = 1/\sqrt{2} = \sqrt{2}/2$

$\cos 3\pi/4 = -1/\sqrt{2} = -\sqrt{2}/2$

$r = 8\sin 3\theta$

$r' = 24\cos 3\theta$

$$\frac{dy}{dx} = \frac{8\sin 3\theta\cos\theta + 24\cos 3\theta\sin\theta}{-8\sin 3\theta\sin\theta + 24\cos 3\theta\cos\theta} = \frac{8\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + 24\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}{-8\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + 24\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}$$

$$= \frac{4-12}{-4-12} = \frac{-8}{-16} = \boxed{\frac{1}{2}}$$