Stat 2470, 2/10 Discussion Questions Name ______________________________

Instructions: Attempt to answer these questions by reading the textbook or with online resources before coming to class on the date above.

1. What is the exponential distribution function?

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

2. The exponential distribution also models part of a Poisson process. How does it differ from the Poisson random variable? Which part of the process does each model?

Poisson models # of events per unit of time; exponential models time until the next event.

Sometimes the Poisson formula also uses λ instead of μ as parameter.

3. What is the expected value and variance of the exponential distribution?

$$E(X) = \int_0^\infty x \lambda e^{-\lambda x} dx = \frac{1}{\lambda}$$

4. What do we mean when we say a process is "memoryless"?

the time (expected) until the next event does not depend on how much time has already passed since the last one.

5. What is the gamma function? (Not the gamma distribution.)

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$

6. What are some common gamma function values? What rules can we use for calculating gamma function values for some common cases? Use these rules to find the values for $\Gamma(6)$ and $\Gamma\left(\frac{7}{2}\right)$.

$$\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$$

for $\alpha = n$ (in $\mathbb{N}$) $\Gamma(n) = (n-1)!$

$$\Gamma(1/2) = \sqrt{\pi}$$

7. What is the formula for the gamma distribution? How does the standard gamma distribution differ?

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

for the standard gamma distribution $\beta = 1$

8. What is another name for the standard gamma distribution?

incomplete gamma function

9. What is the expected value and variance of the gamma distribution?

$E(X) = \alpha\beta$ $\quad$ $V(X) = \alpha\beta^2$

10. What is the function for the χ^2 distribution? In what way is it like a special case of the gamma distribution? What are α and β?

$$f(x;\nu) = \begin{cases} \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{(\nu/2)-1} e^{-x/2} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$\alpha = \nu/2, \quad \beta = 2$

11. The χ^2 distribution is in our calculator. What is the syntax for this function? (We will use it again in Chapter 14.)

χ^2cdf(lower, upper, degrees of freedom)
(ν) → Greek letter nu not English vee

12. How is a joint probability mass (density) function different from a (single variable) probability mass function?

the function depends on 2 (or more) variables

13. In the discrete case, $\sum_x \sum_y p(x,y)$ is equal to what?

1

14. Interpret in words the mathematical statement $p(x, y) = P(X = x, Y = y)$.

the probability when both $X = x$ and $Y = y$

15. Discrete probability mass functions are often displayed as a table. Give the discrete probability mass function for the joint event "flipped a fair coin (X) and rolling a fair die (Y)".

X = coin \ Y = dice →	1	2	3	4	5	6
H	1/12	1/12	1/12	1/12	1/12	1/12
T	1/12	1/12	1/12	1/12	1/12	1/12

16. How do the marginal probability mass functions relate ~~to the joint probability mass functions relate~~ to the joint probability mass function in the discrete case?

$P_X(x) = \sum_y p(x,y)$ for each value of x (can display in a table)

$P_Y(y) = \sum_x p(x,y)$ for each value of y (ditto)

To discuss the continuous case, we need to quickly review (crash course) the necessary multivariable calculus.

17. How do we find partial derivatives of functions in two variables in your own words?

take the derivative, but pretend all other variables are fixed & treat them like constants, except the one whose derivative you are taking

18. How do you compute a multiple integral? For instance, explain the steps to compute $\int_0^1 \int_0^3 x^2 y\,dy\,dx$?

first integrate $y\,dy$ and then plug in 3 and 0. next use the result to integrate the product of that and $x^2\,dx$ then plug in 1 and 0 to obtain the result.

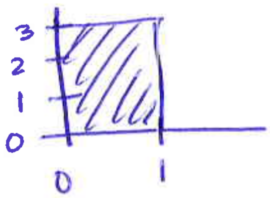
$$\int_0^1\int_0^3 x^2 y\,dy\,dx = \int_0^1 x^2 \cdot \frac{y^2}{2}\Big|_0^3 dx = \int_0^1 \frac{9}{2}x^2\,dx = \frac{9}{2}\cdot\frac{x^3}{3}\Big|_0^1 = \frac{3}{2}$$

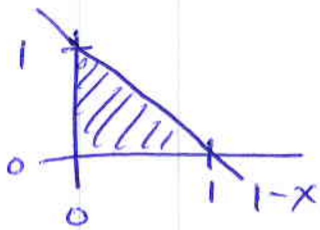
19. How does this change if we want $\int_0^1 \int_0^{1-x} x^2 y\,dy\,dx$? it doesn't very much, only we replace 3 w/ $1-x$ and then we'll need to do some algebra before integrating x.

$$\int_0^1\int_0^{1-x} x^2 y\,dy\,dx = \int_0^1 x^2\,\frac{y^2}{2}\Big|_0^{1-x} dx = \int_0^1 \frac{1}{2}x^2(1-x)^2\,dx = \frac{1}{2}\int_0^1 x^2(1-2x+x^2)\,dx =$$

$$\frac{1}{2}\int_0^1 x^2 - 2x^3 + x^4\,dx = \frac{1}{2}\left[\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5}\right]_0^1 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right] = \frac{1}{2}\left[\frac{1}{30}\right] = \frac{1}{60}$$

20. Explain the region A in the xy-plane described by the integrals in #18 and #19? Sketch them.





21. Joint probability density functions also exist in the continuous case. What must $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dydx$ equal?

1

22. How can we find the value of k needed to make $\int_0^1\int_0^3 kx^2ydydx$ a probability function? What about the k needed for $\int_0^1\int_0^{1-x} kx^2ydydx$?

integrate & set result = 1

$k\int_0^1 x^2 \frac{y^2}{2}\Big|_0^{1-x} dx = \frac{k}{2}\int_0^1 x^2(1-x)^2 dx = \frac{k}{2}\int_0^1 x^2(1-2x+x^2)dx$

$= \frac{k}{2}\int_0^1 x^2-2x^3+x^4\,dx = \frac{k}{2}\left[\frac{x^3}{3}-\frac{2x^4}{4}+\frac{x^5}{5}\right]_0^1 = \frac{k}{2}\left[\frac{1}{3}-\frac{1}{2}+\frac{1}{5}\right] =$

$\frac{k}{2}\left[\frac{1}{30}\right] = \frac{k}{60} = 1 \Rightarrow \boxed{k=60}$

$k\int_0^1\int_0^3 x^2y\,dy\,dx = k\cdot\frac{3}{2} = 1 \Rightarrow k = \frac{2}{3}$

23. How do we find the marginal probability density functions $f_X(x)$ and $f_Y(y)$?

$f_X = \int_{-\infty}^{\infty} f(x,y)\,dy \qquad f_Y = \int_{-\infty}^{\infty} f(x,y)\,dx$

integrate over the variable you wish to eliminate

24. What conditions need to be satisfied for continuous and discrete joint probability density functions to be independent?

$f_X(x)\cdot f_Y(y) = f(x,y)$ for all x, y

or $P_X(x)\cdot P_Y(y) = p(x,y)$