Stat 2470, 2/3 Discussion Questions Name _____

Instructions: Attempt to answer these questions by reading the textbook or with online resources before coming to class on the date above.

1. How do we calculate the probability for a continuous probability distribution?

$P(a \le X \le b) = \int_a^b f(x)\,dx$ w/ $f(x)$ the probability density function

2. Why is $P(X = a)$ always zero for a continuous density function?

The area $\int_a^a f(x)dx = 0$

3. What must $\int_{-\infty}^{\infty} f(x)dx$ equal if it is a legitimate probability density function?

1 (The sum of all possible probabilities always equals one)

4. What is the formula for a uniform probability density function?

$$f(x;A,B) = \begin{cases} \frac{1}{B-A} & A \le x \le B \\ 0 & \text{otherwise} \end{cases}$$

5. Suppose that you are given a probability density function given by $f(x) = \begin{cases} kx^3, 0 \le x \le 1 \\ 0, \quad otherwise \end{cases}$. How can we find the value of k to make this a legitimate probability density function?

integrate and set result = 1.

$k\int_0^1 x^3 dx = \frac{k}{4}x^4\Big|_0^1 = \frac{k}{4} = 1 \Rightarrow k = 4$ $\quad f(x) = \begin{cases} 4x^3 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

6. How do we convert a probability density function to a cumulative distribution function?

integrate to x $\quad F(x) = \int_{-\infty}^{x} f(y)dy$ for example in #5

$F(x) = \int_0^x 4y^3 dy = x^4$ for x on $[0,1]$

7. How can you use a cumulative distribution function to calculate $P(a \le X \le b)$?

$F(b) - F(a)$

8. Given the cumulative distribution function, how can you find the probability density function from it?

take the derivative

9. What is a percentile? For instance, what do we mean when we talk about the 90th percentile?

a percentile is the value at which the given % has accumulated below that point. So 25th percentile is the x for which $P(X \leq x) = .25$

10. How do we calculate percentiles from probability density functions? From continuous distribution functions?

$\int_{-\infty}^{x} f(y)\,dy$ = percentile as a decimal (.25 for 25th percentile)

then integrate & solve for x.

11. How do we find the median of a continuous distribution function?

find the value of the 50th percentile

12. How do we find the mode (if it exists) of a continuous distribution function?

the maximum of the distribution. find $f'(x) = 0$ & solve for x.

13. What does it mean mathematically for a distribution to be symmetric?

$f(x) = f(-x)$ for all x

or more generally

$f(x+\mu) = f(x-\mu)$ to be symmetric around μ instead of zero