Stat 2470, Quiz #7, Spring 2015 Name KEY

Instructions: Show all work. State any formulas used. If you use the calculator, you should say which function you used, and what you entered into it, as well as any output. I can only give partial correct for incorrect answers if I have something to grade.

1. For the joint probability distribution $f(x,y) = \begin{cases} 24x^2y^4, 0 \le x \le 1, x \le y \le 1 \\ 0, \quad\quad otherwise \end{cases}$, find $E(X)$, and $E(XY)$.

$$E(x) = \int_0^1 \int_x^1 24x^3y^4\,dy\,dx = \int_0^1 \frac{24}{5}x^3y^5\Big|_x^1 dx = \int_0^1 \frac{24}{5}x^3(1-x^5)dx$$

$$= \int_0^1 \frac{24}{5}(x^3-x^8)dx = \frac{24}{5}\left[\frac{x^4}{4}-\frac{x^9}{9}\right]_0^1 = \frac{24}{5}\left[\frac{1}{4}-\frac{1}{9}\right] = \frac{24}{5}\left[\frac{5}{36}\right] = \frac{2}{3}$$

$$E(xy) = \int_0^1 \int_x^1 24x^3y^5\,dy\,dx = \int_0^1 \frac{24}{6}x^3y^6\Big|_x^1 dx = \int_0^1 4x^3(1-x^6)dx =$$

$$4\int_0^1 x^3 - x^9\,dx = 4\left[\frac{x^4}{4}-\frac{x^{10}}{10}\right]_0^1 = 4\left(\frac{1}{4}-\frac{1}{10}\right) = 4\left(\frac{3}{20}\right) = \frac{3}{5}$$

2. A discrete joint probability mass function is shown in the table below.

$f(x,y)$		y = 0	1	2	3	4	5
x	0	0.02	0.05	0.07	0.11	0.14	0.19
	1	0.18	0.09	0.06	0.04	0.03	0.02

a. Find $E(Y)$

	0	1	2	3	4	5
$f_y(y)=$	.2	.14	.13	.15	.17	.21

$$E(y) = 0(.2) + 1(.14) + 2(.13) + 3(.15) + 4(.17) + 5(.21) = 2.58$$

b. Find $f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)}$ for $X = 0$.

$f_X(0) = .02 + .05 + .07 + .11 + .14 + .19 = .58$

Y	0	1	2	3	4	5
$f_{Y\|X}$	$.02/.58$	$.05/.58$	$.07/.58$	$.11/.58$	$.14/.58$	$.19/.58$
	$=.0344$	$=.0862$	$=.120689$	$=.189655$	$=.241379$	$=.327586$
	$1/29$	$5/58$	$7/58$	$(11/58)$	$7/29$	$19/58$

3. What is the importance of the Central Limit Theorem?

it allows us to estimate parameters using samples and the normal distribution and lets us give an estimate for how likely our estimate is to be a good estimate

4. Under what circumstances do we apply the principle of minimum variance unbiased estimation (MVUE)?

When there are multiple unbiased estimators for a parameter, this principle tells us to choose the one w/ the least variance as its more likely to give us a better estimate.