

**Instructions:** Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Consider the following matrices:  $A = \begin{bmatrix} 1 & -3 \\ 4 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & -4 \end{bmatrix}$ ,  $C = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 5 \\ -1 & 1 & 3 \end{bmatrix}$ , find each of the following matrices or say that they are undefined. If they are undefined, explain why.

a.  $2A^T - 5I_2$   $A^T = \begin{bmatrix} 1 & 4 \\ -3 & 1 \end{bmatrix}$   $2A^T = \begin{bmatrix} 2 & 8 \\ -6 & 2 \end{bmatrix}$   $-5I_2 = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$

$$2A^T - 5I_2 = \begin{bmatrix} 2 & 8 \\ -6 & 2 \end{bmatrix} + \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} -3 & 8 \\ -6 & -3 \end{bmatrix}$$

b.  $BC$   $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & -4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 5 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1+0+1 & -1+0+1 & 2+0+3 \\ 2+2+4 & -2+0-4 & 4+5-12 \end{bmatrix} =$   
 $\begin{bmatrix} 2 & 0 & 6 \\ 8 & -6 & -3 \end{bmatrix}$

c.  $BA$   $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & -4 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 4 & 1 \end{bmatrix}$  not defined  
 interior dimensions don't match

2. Use an inverse matrix to solve the system  $\begin{cases} 7x_1 + 2x_2 = -9 \\ -6x_1 - 2x_2 = 4 \end{cases}$

$$A = \begin{bmatrix} 7 & 2 \\ -6 & -2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} -9 \\ 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-14+12} \begin{bmatrix} -2 & -2 \\ 6 & 7 \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} -2 & -2 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -7/2 \end{bmatrix}$$

$$A^{-1}\vec{b} = \begin{bmatrix} 1 & 1 \\ -3 & -7/2 \end{bmatrix} \begin{bmatrix} -9 \\ 4 \end{bmatrix} = \begin{bmatrix} -9+4 \\ 27-14 \end{bmatrix} = \begin{bmatrix} -5 \\ 13 \end{bmatrix}$$