

Instructions: Show all work to receive full credit. You should note any formulas used or calculator functions used, their inputs and outputs. I cannot grade work if I don't know where an answer came from. Be sure complete all parts of each questions, including requests for interpretation and explanations. Be as thorough as possible.

1. The 100 tiles in Scrabble are distributed as follows:

Tile	Number	Tile	Number	Tile	Number	Tile	Number
Blank	2	A	9	B	2	C	2
D	4	E	12	F	2	G	3
H	2	I	9	J	1	K	1
L	4	M	2	N	6	O	8
P	2	Q	1	R	6	S	4
T	6	U	4	V	2	W	2
X	1	Y	2	Z	1		

- a. What is the probability of selecting an R as the first tile in a game? (4 points)

$$\frac{6}{100} = \frac{3}{50} = 6\%$$

- b. What is the probability of selecting a vowel (not Y) as the first tile? (5 points)

$$9 + 12 + 9 + 8 + 4 = 42$$

$$\frac{42}{100} = \frac{21}{50} = 42\%$$

- c. What is the probability of not selecting a vowel? (4 points)

$$\frac{58}{100} = \frac{29}{50} = 58\%$$

- d. What is the probability of selecting the word BOX in order from the first three tiles in a game? (6 points)

$$\frac{2}{100} \cdot \frac{8}{99} \cdot \frac{1}{98} = 1.649 \times 10^{-5}$$

2. Use the following table to calculate the probabilities requested. (5 points each)

		STUDENT'S CHOICE		Total
		Art degree	Science degree	
GROUP	Boys	25	50	75
	Girls	55	20	75
Total		80	70	150

- a. What is the probability of a randomly selected person from this study is a girl?

$$\frac{75}{150} = 50\%$$

- b. What is the probability of a randomly selected person from this study being a girl and wants a science degree?

$$\frac{20}{150} = \frac{2}{15} \approx 13.3\%$$

- c. What is the probability of a randomly selected person from this study being a girl or wanting a science degree?

$$\frac{75}{150} + \frac{70}{150} - \frac{20}{150} = \frac{125}{150} = \frac{5}{6} \approx 83.3\%$$

- d. What is the probability of being a girl given that the person wants a science degree?

$$\frac{20}{70} = \frac{2}{7} \approx 28.6\%$$

- e. Are the variables gender and degree choice independent? Why or why not? Show calculations to justify your answer.

They are not independent since the answers to (a) and (d) are not equal

3. Determine the **number** of outcomes in each of the following scenarios. (6 points each)
- a. A local area network requires eight characters for a username and is not case sensitive, but the character can use numbers in the last three digits. How many usernames of this type are there?

$$26^5 36^3 = 5.54 \times 10^{11}$$

- b. Suppose that a lottery has 42 balls, and someone needs 6 matches in any order to win the top prize. How many possible winning number combinations are possible?

$$42C6 = 5,245,786$$

- c. Suppose that ten horses are in a particular race. How many ways can the top three horses finish?

$$10P3 = 720$$

- d. Tim is planning to create a word scramble puzzle out of the word ENCYCLOPEDIA. How many different sequences of the letters are possible?

$$\frac{12!}{2!2!} = 119,750,400$$

4. Find the probability of each of the following scenarios. (6 points each)
- a. Social Security numbers are composed of 9 digits, and each digit can be any number from 0-9. What is the probability that a randomly selected Social Security number will be all even numbers?

$$\frac{5^9}{10^9} = \left(\frac{5}{10}\right)^9 = \left(\frac{1}{2}\right)^9 = .001953$$

$$= \frac{1}{512}$$

- b. What is the probability of getting a three-of-a-kind in a 5-card poker hand?

$$\frac{(13)(4C3)(48C2)}{52C5} = .022569...$$

5. Evaluate the following expressions. (3 points each)

a. $\binom{9}{4} = 126$

b. ${}^7P_3 = 210$

c. ${}^{12}C_5 = 792$

d. $0! = 1$

6. Use the following table to find the probability of getting the sum of 14 on rolling two 8-sided dice. (6 points)

Sums	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

$$\frac{3}{64}$$

7. A certain game is played by drawing cards with numbers on them according to the following distribution. (8 points)

Score, x	Probability, $P(x)$
0	0.07
1	0.13
2	0.18
3	0.30
4	0.22
5	0.08
6	0.02
$\sum P(x) = 1.00$	

Find the expected value (mean) and the standard deviation of this card game.

1 VarStats L1, L2

$$\bar{x} = 2.79$$

$$\sigma_x = 1.416$$

8. A charity sells 450 tickets for a raffle, costing \$10 per ticket. The top prize is \$500, a second prize of \$100, and two third prizes of \$50. For someone purchasing a ticket, what is the expected value? Interpret the value in the context of the problem. (8 points)

Value	490	90	40	-10
Probability	$\frac{1}{450}$	$\frac{1}{450}$	$\frac{2}{450}$	$\frac{446}{450}$

$$\frac{490}{450} + \frac{90}{450} + \frac{40}{450} - 10 \left(\frac{446}{450} \right) = \$ -8.44$$

for each ticket purchased one can expect to lose \$8.44 on average

9. Two thirds of drivers put their seat belt on when they get into a driving simulator. Use that fact to answer the following questions. (5 points each)
- a. If 10 people get into the simulator, what is the probability that exactly 8 of the people will put on their seat belts?

$$n=10, p=\frac{2}{3}$$

$$\text{binomial pdf}(10, \frac{2}{3}, 8) = .19509$$

$$19.5\%$$

- b. If 10 people get into the simulator, what is the probability that no more than 8 people will put on their seat belts?

$$0.12345678 | 910$$

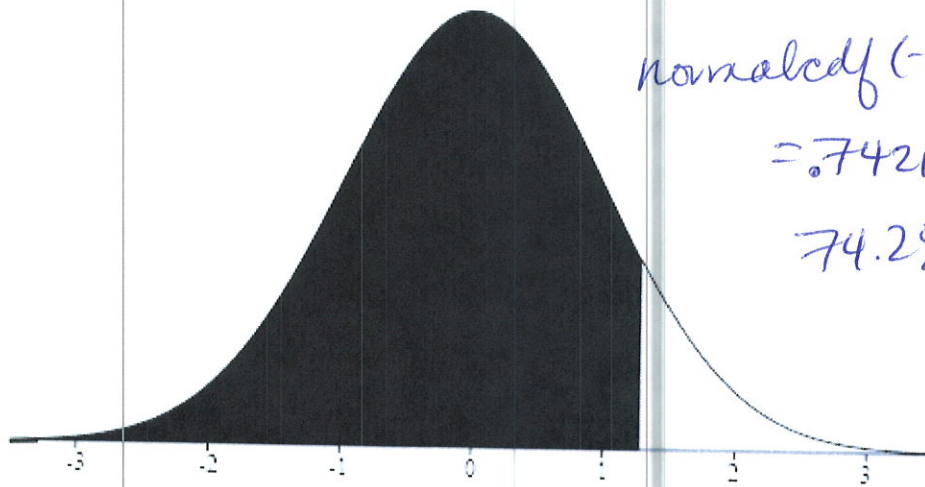
$$\text{binomial cdf}(10, \frac{2}{3}, 8) = .89595$$

$$89.6\%$$

- c. What is the expected number of those 10 people who put on their seat belts?

$$np = 10 \cdot \frac{2}{3} \approx 6.67$$

10. Find the probability under the curve of the given normal distributions. (4 points each)
- a. Standard normal distribution. Z-score at the boundary is 0.65.

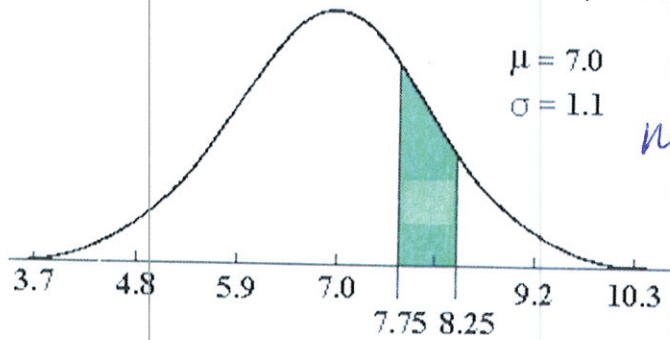


$$\text{Normalcdf}(-E99, 0.65)$$

$$= .7421$$

$$74.2\%$$

- b. Mean is 7.0 and the standard deviation 1.1. The probability between 7.75 and 8.25.



$$\mu = 7.0$$

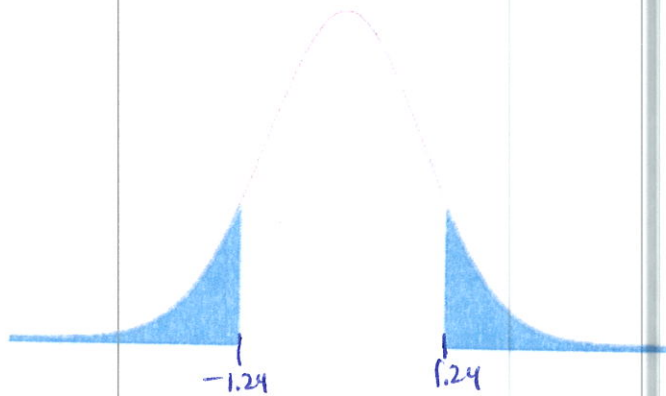
$$\sigma = 1.1$$

$$\text{Normalcdf}(7.75, 8.25, 7, 1.1)$$

$$= .11977$$

$$12.0\%$$

- c. The distribution is the standard normal distribution. The boundaries are ± 1.24 .



$$2 * \text{Normalcdf}(1.24, E99) = .2149755$$

$$21.5\%$$

11. The SAT has a mean score of 1498 and a standard deviation of 199. (5 points each)
- a. What is the z-score of 1640?

$$\frac{1640 - 1498}{199} = .7135678$$

- b. What score represents the 90th percentile of the distribution? Round your answer to the nearest 10 points.

$$\text{invNorm}(.90, 1498, 199) = 1753 \Rightarrow 1750$$

- c. If a school wants to admit only students with the top 5% of SAT scorers, what cut-off score is needed? Round your answer to the nearest 10 points.

$$\text{invNorm}(.95, 1498, 199) = 1825.32 \dots$$
$$\Rightarrow 1830$$

- d. The mean score on the ACT is 21 with a standard deviation of 5.2. Which student scored higher: Abby with a score of 28 on the ACT, or Barbara with a score of 1910 on the SAT?

$$z_A = \frac{28 - 21}{5.2} = 1.346$$

$$z_B = \frac{1910 - 1498}{199} = 2.07$$

Barbara has the higher score

12. What conditions need to be satisfied in order to use the normal distribution to approximate a binomial distribution? (4 points)

$$np > 5$$

and

$$n(1-p) > 5$$