MAT 201, Quiz #11, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Evaluate the surface integral $\iint_S (x+y+z)dS$ for S given by $\vec{r}(u,v) = (u+v)\hat{\imath} + (u-v)\hat{\jmath} + (1+2u+v)\hat{k}, 0 \le u \le 2, 0 \le v \le 1$

$x+y+z = u+\cancel{v} + u-\cancel{v} + 2u+v+1 = 1+4u+v$

$r_u = \hat{\imath}+\hat{\jmath}+2\hat{k} \quad r_v = \hat{\imath}-\hat{\jmath}+\hat{k}$

$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix} = (1+2)\hat{\imath} - (1-2)\hat{\jmath} + (-1-1)\hat{k}$
$3\hat{\imath}+\hat{\jmath}-2\hat{k}$

$\|\vec{r}_u \times \vec{r}_v\| = \sqrt{9+1+4} = \sqrt{14}$

$\int_0^2\int_0^1 (1+4u+v)(\sqrt{14})\,dv\,du$

$= \sqrt{14}\int_0^2 v+4uv+\frac{1}{2}v^2\Big|_0^1 du =$

$\sqrt{14}\int_0^2 1+4u+\frac{1}{2}\,du = \sqrt{14}\int_0^2 \frac{3}{2}+4u\,du$

$= \sqrt{14}\left[\frac{3}{2}u+2u^2\Big|_0^2\right] = \sqrt{14}(3+8) =$ $\boxed{11\sqrt{14}}$

2. Calculate the flux $\iint_S \vec{F}\cdot d\vec{S}$ using the divergence theorem for $\vec{F}(x,y,z) = xye^z\hat{\imath} + xy^2z^3\hat{\jmath} - ye^z\hat{k}$ for S the box bounded by the coordinate planes and $x=3, y=2, z=1$.

$\nabla\cdot F = ye^z + 2xyz^3 - ye^z = 2xyz^3$

$\int_0^3\int_0^2\int_0^1 2xyz^3\,dz\,dy\,dx = \int_0^3\int_0^2 \frac{1}{2}xyz^4\Big|_0^1 dy\,dx = \int_0^3\int_0^2 \frac{1}{2}xy\,dy\,dx =$

$\int_0^3 \frac{1}{4}xy^2\Big|_0^2 dx = \int_0^3 \frac{1}{4}x\cdot 4\,dx = \int_0^3 x\,dx = \frac{1}{2}x^2\Big|_0^3 = \boxed{\frac{9}{2}}$

3. How many surface integrals would be needed to calculate #2 without the divergence theorem?

6