MAT 201, Quiz #12, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Set up the change of coordinates for $\iint_R xy\,dA$ where R bounded by $y = x, y = 3x, xy = 1, xy = 3$. (Let $x = \frac{u}{v}, y = v$.)

$v = y = \frac{u}{v} \Rightarrow v^2 = u \rightarrow v = \sqrt{u}$

$v = y = 3\frac{u}{v} \Rightarrow \frac{1}{3}v^2 = u \quad v = \sqrt{3u}$

$J = \begin{vmatrix} \frac{1}{v} & -\frac{u}{v^2} \\ 0 & 1 \end{vmatrix} = \frac{1}{v} + 0 = \frac{1}{v}$

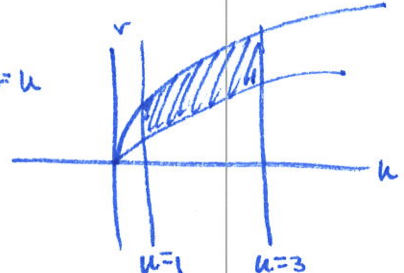
$xy = \frac{u}{v} \cdot v = u = 1$

$xy = \frac{u}{v} \cdot v = u = 3$

$$\int_1^3 \int_{\sqrt{u}}^{\sqrt{3u}} \frac{u}{v}\,dv\,du$$

(you don't need to integrate)

$xy = \frac{u}{v} \cdot v = u$



2. Find the directional derivative for the surface $g(x, y) = \tan^{-1}(xy)$ at (2,1) in the direction $\vec{v} = \hat{\imath} + 2\hat{\jmath}$.

$\nabla g = \left(\frac{1}{1+x^2y^2} \cdot y\right)\hat{\imath} + \left(\frac{x}{1+x^2y^2}\right)\hat{\jmath} \quad \nabla g(2,1) = \left(\frac{1 \cdot 1}{1+4 \cdot 1}\right)\hat{\imath} + \left(\frac{2}{1+4 \cdot 1}\right)\hat{\jmath} = \left(\frac{1}{5}\right)\hat{\imath} + \left(\frac{2}{5}\right)\hat{\jmath}$

$\|\vec{v}\| = \sqrt{1+4} = \sqrt{5}$

$\hat{v} = \frac{1}{\sqrt{5}}\hat{\imath} + \frac{2}{\sqrt{5}}\hat{\jmath}$

$\nabla g(2,1) \cdot \hat{v} = \frac{1}{5} \cdot \frac{1}{\sqrt{5}} + \frac{2}{5} \cdot \frac{2}{\sqrt{5}} = \frac{1}{5\sqrt{5}} + \frac{4}{5\sqrt{5}} = \frac{5}{5\sqrt{5}}$

$= \boxed{\frac{1}{\sqrt{5}}}$

3. In what direction is the directional derivative a maximum for the surface $y = x^2 - z^2$ at the point (4,7,3)?

$z^2 = x^2 - y \Rightarrow z = \sqrt{x^2 - y}$

$\nabla z = \frac{1}{2}(x^2-y)^{-1/2} \cdot 2x\hat{\imath} + \frac{1}{2}(x^2-y)^{-1/2} \cdot (-1)\hat{\jmath}$

$\nabla z = \frac{x}{\sqrt{x^2-y}}\hat{\imath} - \frac{1}{2\sqrt{x^2-y}}\hat{\jmath}$

$\nabla z(4,7) = \frac{4}{\sqrt{16-7}}\hat{\imath} - \frac{1}{2\sqrt{16-7}}\hat{\jmath} = \frac{4}{\sqrt{9}}\hat{\imath} - \frac{1}{2\sqrt{9}}\hat{\jmath} = \boxed{\frac{4}{3}\hat{\imath} - \frac{1}{6}\hat{\jmath}}$

$\sqrt{\frac{16}{9} - \frac{1}{36}} = \sqrt{\frac{64}{36} - \frac{1}{36}} = \sqrt{\frac{63}{36}} = \frac{3\sqrt{7}}{6}$

$\frac{\nabla z}{\|\nabla z\|} = \frac{4}{3} \cdot \frac{6}{3\sqrt{7}}\hat{\imath} - \frac{1}{6} \cdot \frac{6}{3\sqrt{7}}\hat{\jmath} = \boxed{\frac{8}{3\sqrt{7}}\hat{\imath} - \frac{1}{3\sqrt{7}}\hat{\jmath}}$

4. Find the equation of the linear approximation $L(x, y)$ to the function $z = \sqrt{x^2 - y}$ at the point $(4,7,3)$ and use it to estimate the value of z at the point $(4.1, 6.95, z + \Delta z)$.

$$\nabla F = \frac{x}{\sqrt{x^2-y}}\,\hat{\imath} - \frac{1}{2\sqrt{x^2-y}}\,\hat{\jmath} - \hat{k}$$

$$\nabla F(4,7,3) = \frac{4}{3}\hat{\imath} - \frac{1}{6}\hat{\jmath} - 1\hat{k} \qquad \langle \tfrac{4}{3}, \tfrac{-1}{6}, -1 \rangle$$

$$\frac{4}{3}(x-4) - \frac{1}{6}(y-7) - 1(z-3) = 0$$

$$\frac{4}{3}x - \frac{16}{3} - \frac{1}{6}y + \frac{7}{6} - z + 3 = 0$$

$$\boxed{L(x,y) = \frac{4}{3}x - \frac{1}{6}y - \frac{7}{6}}$$

$$L(4.1, 6.95) = \frac{4}{3}(4.1) - \frac{1}{6}(6.95) - \frac{7}{6} = \boxed{3.1416\overline{6} \text{ or } \frac{377}{120}}$$

$$\Delta z \approx .1416\overline{6} \text{ or } \frac{17}{120}$$