

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Rewrite each equation in standard form and sketch the surface.

a. $y^2 = x^2 - \frac{1}{9}z^2$

$$y^2 + \frac{1}{9}z^2 = x^2 \quad \text{Cone wrapped around } x \text{ axis}$$

b. $x^2 - y^2 + z^2 - 4x - 2y - 2z + 4 = 0$

$$(x^2 - 4x + 4) - (y^2 + 2y + 1) + (z^2 - 2z + 1) = -4 + 4 - 1 + 1 = 0$$

$$(x-2)^2 - (y+1)^2 + (z-1)^2 = 0$$

$$(x-2)^2 + (z-1)^2 = (y+1)^2$$

cone wrapped around "y-axis" orientation
w/ vertex at $(2, -1, 1)$

c. $4y^2 + z^2 - x - 16y - 4z + 20 = 0$

$$4(y^2 - 4y + 4) + (z^2 - 4z + 4)$$

$$4(y-2)^2 + (z-2)^2 = x$$

$$-20 + x + 16 + 4$$

paraboloid opening around positive x
axis vertex at $(0, 2, 2)$.

2. The equation $x^2 + y^2 + z^2 = 16$ is a sphere. Write the equation in:

- a. Cylindrical coordinates

$$r^2 + z^2 = 16$$

- b. In spherical coordinates

$$\rho = 4$$

- c. As a parametric surface

$$4 \cos u \cos v \hat{i} + 4 \sin u \cos v \hat{j} + 4 \sin v \hat{k}$$

$$0 \leq u \leq 2\pi$$

$$0 \leq v \leq \pi$$

3. For the equation $x^2 + y^2 = z^2$, rewrite the equation in:
- Cylindrical coordinates

$$r = z$$

- Spherical coordinates

$$\rho^2 \sin^2 \varphi = \rho^2 \cos^2 \varphi$$

$$\tan \varphi = 1 \Rightarrow \varphi = \pi/4$$

- As a parametric surface

$$r = z = u$$

$$\theta = v$$

$$u \cos v \hat{i} + u \sin v \hat{j} + u \hat{k}$$

- What kind of surface is this?

Cone