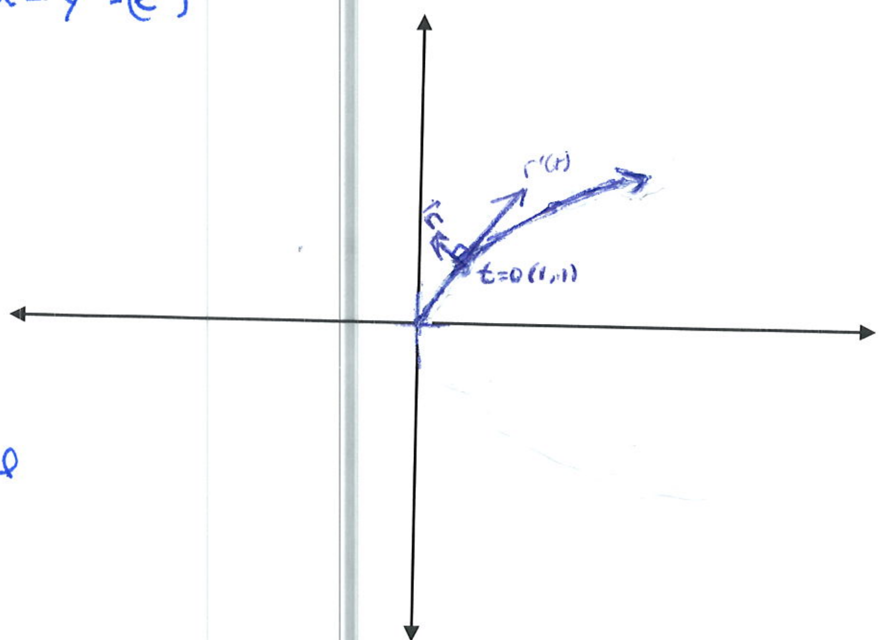
MAT 201, Quiz #5, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Sketch the curve given by $\vec{r}(t) = e^{2t}\hat{\imath} + e^{t}\hat{\jmath}$.

$y = e^t > 0$
$x = y^2 = (e^t)^2$



a. Find the derivative $\vec{r}'(t)$.

$r'(t) = 2e^{2t}\hat{\imath} + e^{t}\hat{\jmath}$

b. Sketch the tangent vector and the normal vector on the graph at the point $t = 0$.

(inward or outward normal okay)

c. Write the equation of the tangent line at the same point.

$r'(0) = 2\hat{\imath} + 1\hat{\jmath}$ $\langle 2,1\rangle$ pt $(1,1)$

$t(t) = (2t+1)\hat{\imath} + (t+1)\hat{\jmath}$
tangent line

2. For the space curve $\vec{r}(t) = e^{t}\hat{\imath} + te^{t}\hat{\jmath} + te^{t^2}\hat{k}$, find:

a. $\vec{r}'(t)$

$e^t\hat{\imath} + (e^t + te^t)\hat{\jmath} + (e^{t^2} + 2t^2e^{t^2})\hat{k}$

$\boxed{e^t\hat{\imath} + e^t(1+t)\hat{\jmath} + e^{t^2}(1+2t^2)\hat{k}} \rightarrow \boxed{1\hat{\imath} + 1\hat{\jmath} + 1\hat{k}}\ e.$

b. $\vec{r}''(t)$

$e^t\hat{\imath} + [e^t(1+t) + e^t(1)]\hat{\jmath} + [2te^{t^2}(1+2t^2) + e^{t^2}(4t)]\hat{k}$

$\boxed{e^t\hat{\imath} + e^t(2+t)\hat{\jmath} + e^{t^2}(6t+4t^3)\hat{k}} \rightarrow \boxed{1\hat{\imath} + 2\hat{\jmath} + 0\hat{k}}\ e$

c. $\vec{r}'(t) \times \vec{r}''(t)$

See last page

d. $\vec{r}'(t) \cdot \vec{r}''(t)$

$e^{2t} + e^{2t}\underbrace{(1+t)(2+t)}_{t^2+3t+2} + e^{2t^2}\underbrace{(1+2t^2)(6t+4t^3)}_{6t+4t^3+12t^3+8t^5} = \boxed{e^{2t}(t^2+3t+3) + e^{2t^2}(8t^5+16t^3+6t)}$

$= 3\ (e)$

e. Evaluate each at the point $(1,0,0)$. $t=0$

3. Evaluate $\int \left(te^{2t}\hat{\imath} + \frac{t}{1-t}\hat{\jmath} + \frac{1}{\sqrt{1-t^2}}\hat{k}\right) dt$

$\left(\frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} + C_1\right)\hat{\imath} +$
$\left(-t - \ln|t-1| + C_2\right)\hat{\jmath}$
$+ \left(\sin^{-1}(t) + C_3\right)\hat{k}$

$\int te^{2t}dt$ $\quad u = t \quad dv = e^{2t}$
$du = dt \quad v = \frac{1}{2}e^{2t}$
$\frac{1}{2}te^{2t} - \int \frac{1}{2}e^{2t}dt =$

$\int \frac{t}{1-t}dt = -\int \frac{t}{t-1}$

$t-1\overline{)t}$ $= 1 + \frac{1}{t-1}$
$-\ t+1$
1

$\int -1 - \frac{1}{t-1}\, dt =$
$-t - \ln(t-1) + C$

$\int \frac{1}{\sqrt{1-t^2}}dt = \arcsin t + C$

4. Find the indicated partial derivative(s).

a. $z = \tan(xy)$, z_x

$\sec(xy) \cdot y$

b. $f(x,y) = x^4y^3 + 8x^2y, f_y$

$3x^4y^2 + 8x^2$

c. $w = ze^{xyz}, \frac{\partial w}{\partial z}$

$e^{xyz} + z \cdot e^{xyz} \cdot xy \quad = e^{xyz}(1 + xyz)$

d. $f(x,y) = \sin(2x + 5y), f_{yxy}, f_{xyy}$ (What do you notice?)

$f_y = \cos(2x+5y) \cdot 5$
$f_{yx} = -\sin(2x+5y) \cdot 5 \cdot 2 = -10\sin(2x+5y)$
$f_{yxy} = -10\cos(2x+5y) \cdot 5 = -50\cos(2x+5y)$

$f_x = \cos(2x+5y) \cdot 2$
$f_{xy} = -\sin(2x+5y) \cdot 2 \cdot 5 = -10\sin(2x+5y)$
$f_{xyy} = -10\cos(2x+5y) \cdot 5 = -50\cos(2x+5y)$

f_{yx} and f_{xy} are the same
as are f_{yxy} and f_{xyy}

2c.

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ e^t & e^t(1+t) & e^{t^2}(1+2t^2) \\ e^t & e^t(2+t) & e^{t^2}(6t+4t^3) \end{vmatrix} =$$

$$\left(e^{t^2+t}\left[(1+t)(6t+4t^3) - (2+t)(1+2t^2)\right]\right)\hat{\imath} - e^{t^2+t}\left[(6t+4t^3) - (1+2t^2)\right]\hat{\jmath} +$$

$$e^{2t}\left[(2+t - (1+t)\right]\hat{k} =$$

$$\boxed{e^{t^2+t}(4t^4+2t^3+2t^2+5t-2)\hat{\imath} + e^{t^2+t}(1-6t+2t^2-4t^3)\hat{\jmath} + e^{2t}\hat{k}}$$

$$(1+t)(6t+4t^3) - (2+t)(1+2t^2)$$

$$= 6t+4t^3+6t^2+4t^4 - (2+4t^2+t+2t^3)$$

$$= -2+5t+2t^2+2t^3+4t^4$$

$t=0$

e. $-2\hat{\imath} + \hat{\jmath} + \hat{k}$

$$-\left[\left((6t^4+4t^3\right) - (1+2t^2)\right] =$$

$$-(4t^3 - 2t^2 + 6t - 1)$$

$$= 1 - 6t + 2t^2 - 4t^3$$

$$(2+t) - (1+t)$$

$$= 2+t-1-t = 1$$