MAT 212, Quiz #10, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Determine if $\vec{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \vec{y} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ are orthogonal.

$1+0-1=0$ Yes, they are orthogonal since their dot product is 0

2. Find the Laplace transform of $f(t) = t$ **using the definition**. Recall $L(f) = \int_0^\infty e^{-st} f(t)dt$.

$\int_0^\infty e^{-st} \cdot t\, dt$ $\quad u=t \quad dv = e^{-st}$ $\quad du = dt \quad v = -\frac{1}{s}e^{-st}$

$\frac{-1}{s}te^{-st} - \int_0^\infty -\frac{1}{s}e^{-st}dt = -\frac{t}{s}e^{-st} + \frac{1}{s}\int_0^\infty e^{-st}dt = \frac{-t}{s}e^{-st} - \frac{1}{s^2}e^{-st}\Big|_0^\infty$

$0-0+0+\frac{1}{s^2}e^0 = \boxed{\frac{1}{s^2}}$

3. Use the table to find the inverse Laplace transform of $F(s) = \frac{1-2s}{s^2+4s+5}$. $\frac{1-2s}{s^2+4s+4+1} = \frac{1-2s}{(s+2)^2+1} =$

$\frac{5-2(s+2)}{(s+2)^2+1} = \frac{5}{(s+2)^2+1} - 2\left(\frac{s+2}{(s+2)^2+1}\right)$ $\quad 1-2(s+2)+4 = 5-2(s+2)$

$f(t) =$

$5e^{-2t}\sin t - 2e^{-2t}\cos t$

4. Use Laplace transforms to solve $y'' - 2y' + 2y = \cos t, y(0) = 1, y'(0) = 0$.

$$s^2F(s) - s(1) - 0 - 2(sF(s) - 1) + 2F(s) = \frac{s}{s^2+1}$$

$$s^2F(s) - s - 2sF(s) - 2 + 2F(s) = \frac{s}{s^2+1}$$

$$F(s)(s^2 - 2s + 2) - s - 2 = \frac{s}{s^2+1}$$

$$F(s)(s^2 - 2s + 2) = \frac{s}{s^2+1} + (s+2)\frac{(s^2+1)}{s^2+1} = \frac{s + s^3 + s + 2s^2 + 2}{(s^2+1)}$$

$$F(s) = \frac{s^3 + 2s^2 + 2s + 2}{(s^2+1)(s^2 - 2s + 2)}$$

$$\frac{As+B}{s^2+1} + \frac{Cs+D}{s^2-2s+2} = \frac{(As+B)(s^2-2s+2) + (Cs+D)(s^2+1)}{(s^2+1)(s^2-2s+2)}$$

$$\frac{As^3 - 2As^2 + 2As + Bs^2 - 2Bs + 2B + Cs^3 + Cs + Ds^2 + D}{(s^2+1)(s^2-2s+2)}$$

$(A+C)s^3 = 1s^3$ $\qquad A + C = 1$

$(-2A+B+D)s^2 = 2$ $\qquad -2A + B + D = 2$

$(2A-2B+C)s = 2$ $\qquad 2A - 2B + C = 2$

$(2B+D) = 2$ $\qquad 2B + D = 2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 1 \\ -2 & 1 & 0 & 1 & 2 \\ 2 & -2 & 1 & 0 & 2 \\ 0 & 2 & 0 & 1 & 2 \end{array}\right]$$

$A = 1/5$, $B = -2/5$

$C = 4/5$, $D = 14/5$

$4(s-1) + 18$

$$F(s) = \frac{1}{5}\left(\frac{s+2}{s^2+1}\right) + \frac{1}{5}\left(\frac{4s+14}{s^2-2s+2}\right) = \frac{1}{5}\left(\frac{s}{s^2+1}\right) + \frac{1}{5}\left(\frac{2}{s^2+1}\right) + \frac{1}{5}\left(\frac{4(s-1)}{(s-1)^2+1}\right) + \frac{1}{5}\left(\frac{18}{(s-1)^2+1}\right)$$

$s^2 - 2s + 1 + 1$

$(s-1)^2 + 1$

$$\boxed{y(t) = \frac{1}{5}\cos t + \frac{2}{5}\sin t + \frac{4}{5}e^t\cos t + \frac{18}{5}e^t\sin t}$$

Laplace transforms – Table			
$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t - \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2+\omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t - \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2+\omega^2}$
$\frac{1}{2}t^2e^{-at}$	$\frac{1}{(s+a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$	$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2-\omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1+at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1,2,3..$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
t^ne^{at}	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s}F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t)$ *unit impulse*	$1 \quad$ *all s*
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2f}{df^2}$	$s^2F(s) - sf(0) - f'(0)$
$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{n-1}(0)$		

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\ n=1,2,3,\ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\ n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			