MAT 212, Quiz #12, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Rewrite $u'' + \frac{1}{2}u' + 2u = 0$ as a system of first order equations.

$u = x_1 \quad u' = x_1' = x_2$

$x_2' = u''$

$x_2' + \frac{1}{2}x_2 + 2x_1 = 0$

$x_1' = x_2$

$$\begin{cases} x_1' = x_2 \\ x_2' = -2x_1 - \frac{1}{2}x_2 \end{cases}$$

2. Rewrite the system $\begin{cases} x_1' = 3x_1 - 2x_2 \\ x_2' = 2x_1 - 2x_2 \end{cases}$ as a single second order equation.

$\frac{x_1' - 3x_1}{-2} = x_2$

$-\frac{1}{2}x_1' + \frac{3}{2}x_1 = x_2$

$x_2' = -\frac{1}{2}x_1'' + \frac{3}{2}x_1'$

$-\frac{1}{2}x_1'' + \frac{3}{2}x_1' = 2x_1 - 2(-\frac{1}{2}x_1' + \frac{3}{2}x_1)$

$-\frac{1}{2}x_1'' + \frac{3}{2}x_1' = 2x_1 + x_1' - 3x_1$

$-x_1$

$-2(-\frac{1}{2}x_1'' + \frac{1}{2}x_1' + x_1 = 0)$

$x_1'' - x_1' - 2x_1 = 0$

$\boxed{u'' - u' - 2u = 0}$

3. Find the eigenvalues and eigenvectors of

a. $A = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix}$

$(5-\lambda)(1-\lambda) + 3 = 0$

$5 - 5\lambda - \lambda + \lambda^2 + 3 = 0$

$\lambda^2 - 6\lambda + 8 = 0$

$(\lambda - 4)(\lambda - 2) = 0 \quad \lambda = 4, 2$

$\begin{pmatrix} 3 & -1 \\ 3 & -1 \end{pmatrix}$

$3x_1 - x_2 = 0$

$x_1 = \frac{1}{3}x_2$

$x_2 = x_2$

$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$x_1 - x_2 = 0$

$x_1 = x_2$

$x_2 = x_2$

$\boxed{\begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ for } \lambda = 4 \quad \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ for } \lambda = 2}$

b. $B = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$

$(-2-\lambda)(2-\lambda) - 1 = 0$

$-4 + 2\lambda - 2\lambda + \lambda^2 - 1 =$

$\lambda^2 - 5 = 0$

$\boxed{\lambda = \pm 2\sqrt{5}}$

$\begin{pmatrix} -2+2\sqrt{5} & 1 \\ 1 & 2+2\sqrt{5} \end{pmatrix}$

$x_1 = (-2 \mp 2\sqrt{5})x_2$

$x_2 \quad x_2$

$\boxed{\begin{pmatrix} -2-2\sqrt{5} \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -2+2\sqrt{5} \\ 1 \end{pmatrix}}$

4. Solve $\vec{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}\vec{x}, \vec{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ [Hint: see #3a]

$c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{4t} + c_2\begin{pmatrix} 1 \\ 3 \end{pmatrix}e^{2t} = \vec{x}(t)$

$c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$c_1 = 7/2, \; c_2 = -3/2$

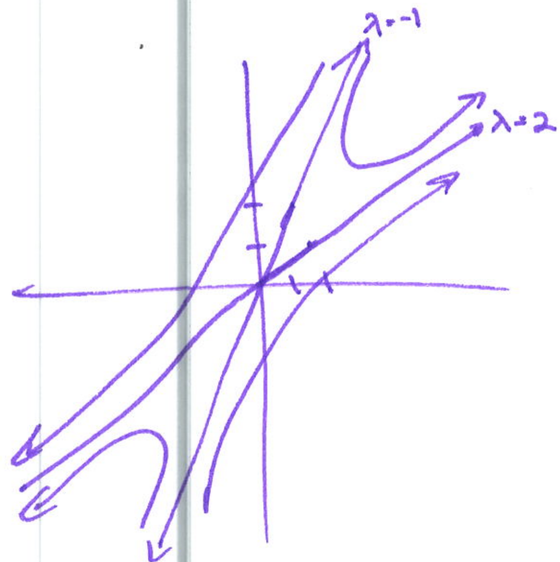
$\vec{x}(t) = \frac{7}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{4t} - \frac{3}{2}\begin{pmatrix} 1 \\ 3 \end{pmatrix}e^{2t}$

5. Draw a direction field for the system $\vec{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \vec{x}$ and plot several trajectories of the system.

$$(3-\lambda)(-2-\lambda)+4=0$$
$$-6-3\lambda+2\lambda+\lambda^2+4=0$$
$$\lambda^2-\lambda-2=0$$
$$(\lambda-2)(\lambda+1)=0$$
$$\lambda=2,\ \lambda=-1$$

$\lambda=2 \begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix}$ $\quad x_1 = 2x_2$, $x_2 = x_2$ $\quad \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\lambda=-1 \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix}$ $\quad 2x_1 = x_2$, $x_1 = \frac{1}{2}x_2$, $x_2 = x_2$ $\quad \begin{pmatrix} 1 \\ 2 \end{pmatrix}$



$$x(t) = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t}$$