MAT 212, Quiz #4, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Rewrite the Bernoulli equation $t^2y' + 2ty - y^3 = 0$ as a linear equation.

$$t^2y' + 2ty = y^3$$

$$-2t^2y^{-3}y' - 4ty^{-2} = -2$$

$$\boxed{t^2z' - 4tz = -2}$$

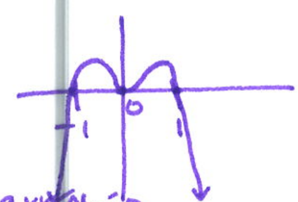
or $\boxed{z' - \frac{4}{t}z = \frac{-2}{t^2}}$

$(1-n)y^{-n} = (-2)y^{-3}$

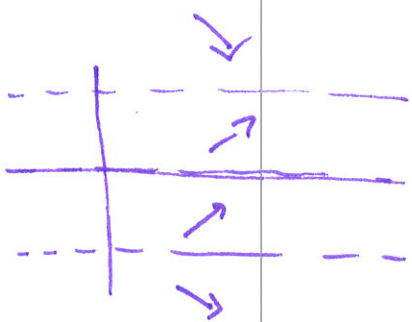
$z = y^{-2}$

$z' = \frac{dz}{dt} = -2y^{-3}y'$

2. Draw the phase plane for the ODE $\frac{dy}{dt} = y^2(1-y^2)$ and use that to characterize each solution as i) stable, unstable or semi-stable; ii) any solution for which $y > 0$ as a threshold, carrying capacity or neither.



$y^2 - y^4$
leading neg.
y=0 repeated root



y=1 stable → carrying capacity

semistable

y=-1 unstable

3. Solve the ODE $(9x^2 + y - 1)dx - (4y - x)dy = 0$.

M N
$-4y+x$

$\frac{\partial M}{\partial y} = 1$ $\frac{\partial N}{\partial x} = 1$ exact

$\int 9x^2 + y - 1\, dx = 3x^3 + xy - x + f(y)$

$\int -4y + x\, dy = -2y^2 + xy + g(x)$

$\psi(x) = xy + 3x^3 - x - 2y^2 + C$

or $xy + 3x^3 - x - 2y^2 = C$

4. Use Euler's method to estimate the solution to the IVP $y' = y(3 - ty), y(0) = 0.5$. If you want to know the value of $y(2)$, and will estimate it using 100 steps, find the first three steps of this calculation. (You should use a minimum of 4 decimal places.)

$h = \Delta t = \frac{2-0}{100} = \frac{2}{100} = \frac{1}{50} = .02$

$y_0 = .5 \quad t_0 = 0$

$m_0 = .5(3 - 0(.5)) = .5(3) = 1.5$

$y_1 = \underset{m}{1.5}(\underset{\Delta t}{.02}) + \underset{.5}{y_0} = .53$

$y_1 = .53 \quad t_1 = .02$

$m_2 = .53(3 - .02(.53)) = 1.554488$

$y_2 = 1.554488(.02) + \underset{=.53}{y_1} \Rightarrow .56108976$

$y_2 = .56108976 \quad t_2 = .04$

$m_3 = .56108976(3 - .04(.56108976)) = 1.670676411$

$y_3 = 1.670676411(.02) + .56108976 = .594503288$

$y_3 \approx .5945 \quad t_3 = .06$

etc.