MAT 212, Quiz #5, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the determinant of $A = \begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & 5 \\ 3 & 1 & -2 \end{pmatrix}$.

$1(0-5)-2(2-15)+4(-1-0)=$

$-5-2(-13)+(-4)$

$-5+26-4=\boxed{17}$

2. Solve the homogeneous second order ODE $y''+4y'+3y=0, y(0)=2, y'(0)=-1$.

$y=e^{rt}$

$r^2+4r+3=0$

$(r+3)(r+1)=0$

$r=1,-3$

$c_1e^{-t}+c_2e^{-3t}=y(t)$ $\quad y'(t)=-c_1e^{-t}-3c_2e^{-3t}$

$c_1+c_2=2$

$-c_1-3c_2=-1$

$0+-2c_2=1$

$c_2=-\frac{1}{2}$

$c_1=\frac{5}{2}$

$\boxed{y(t)=\frac{5}{2}e^{-t}-\frac{1}{2}e^{-3t}}$

3. Find the Wronskian for e^{-2t} and te^{-2t}.

$\begin{vmatrix} e^{-2t} & te^{-2t} \\ -2e^{-2t} & e^{-2t}-2te^{-2t} \end{vmatrix} = e^{-4t}-2te^{-4t}+2te^{-4t}=\boxed{e^{-4t}}$

4. Use Abel's Theorem to find the Wronskian for $t(t-4)y''+3ty'+4y=2$. What is the longest interval on which a solution to the IVP $y(3)=0, y'(3)=-1$ is defined?

$y''+\frac{3\cancel{t}}{\cancel{t}(t-4)}y'+\frac{4}{t(t-4)}y=\frac{2}{t(t-4)}$

$p(t)=\frac{3}{t-4}$

$W=e^{-\int\frac{3}{t-4}dt}=e^{-3\ln(t-4)}=e^{\ln(t-4)^{-3}}=(t-4)^{-3}$

defined for $t\neq4$

IVP defined on $(-\infty, 4)$.

5. Find the general solution to

a. $y'' + 2y' + 5y = 0$

$r^2 + 2r + 5 = 0$

$r = \dfrac{-2 \pm \sqrt{4 - 4 \cdot 5}}{2} = \dfrac{-2 \pm \sqrt{-16}}{2} = \dfrac{-2 \pm 4i}{2} = -1 \pm 2i$

$\boxed{y(t) = c_1 e^{-t} \cos(2t) + c_2 e^{-t} \sin(2t)}$

b. $4y'' + 12y' + 9y = 0$

$4r^2 + 12r + 9 = 0$

$(2r+3)^2 = 0 \quad r = -\frac{3}{2}$ repeated

$\boxed{y(t) = c_1 e^{-3/2 t} + c_2 t e^{-3/2 t}}$