MAT 212, Quiz #7, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find all the complex roots of:

a. $(1+3i)^{\frac{1}{3}}$

$r = 1+9 = \sqrt{10}$ $\left[\sqrt{10}\left(\frac{1}{\sqrt{10}} + \frac{3}{\sqrt{10}}i\right)\right]^{1/3}$

$r^{1/3} = \sqrt[6]{10}$ $\tan^{-1}\left(\frac{3}{1}\right) \approx 1.249$ rads

$\theta_1 = .4163485908$

$\theta_2 = 2.510743693$

$\theta_3 = 4.605138796$

$\approx$ $\boxed{\begin{array}{l} 1.3424 + .593613i \\ -1.1853 + .865753i \\ -.15712 - 1.45937i \end{array}}$

b. $i^{\frac{1}{6}}$

$r = 1$

$\theta = \pi/2$

$\theta_1 = \pi/12$

$\theta_2 = 5\pi/12$

$\theta_3 = 3\pi/4$

$\theta_4 = 13\pi/12$

$\theta_5 = 17\pi/12$

$\theta_6 = 7\pi/4$

$\boxed{\begin{array}{l} \cos \pi/12 + i \sin \pi/12 \\ \cos 5\pi/12 + i \sin 5\pi/12 \\ \cos(3\pi/4) + i\sin(3\pi/4) \\ \cos(13\pi/12) + i \sin(13\pi/12) \\ \cos(17\pi/12) + i\sin(17\pi/12) \\ \cos(7\pi/4) + i \sin(7\pi/4) \end{array}}$

c. $(1+i)^{\frac{1}{4}}$ $r = \sqrt{2}$ $r^{1/4} = \sqrt[8]{2}$

$\theta = \pi/4$

$\theta_1 = \frac{\pi}{16}$

$\theta_2 = 9\pi/16$

$\theta_3 = 17\pi/16$

$\theta_4 = 25\pi/16$

$\boxed{\begin{array}{l} \cos \pi/16 + i \sin \pi/16 \\ \cos 9\pi/16 + i \sin 9\pi/16 \\ \cos 17\pi/16 + i \sin 17\pi/16 \\ \cos 25\pi/16 + i \sin 25\pi/16 \end{array}}$

2. Determine if the set of solutions $f_1(t) = 2t-3, f_2(t) = t^3+1, f_3(t) = 2t^2 - t, f_4(t) = t^2 + t + 1$, are linearly independent.

$$\begin{vmatrix} 2t-3 & t^3+1 & 2t^2-t & t^2+t+1 \\ 2 & 3t^2 & 4t-1 & 2t+1 \\ 0 & 6t & 4 & 2 \\ 0 & 6 & 0 & 0 \end{vmatrix}$$ at $t=0$ $$\begin{vmatrix} -3 & 1 & 0 & 1 \\ 2 & 0 & -1 & 1 \\ 0 & 0 & 4 & 2 \\ 0 & 6 & 0 & 0 \end{vmatrix} \neq 0$$

yes, they are independent

3. Verify that $x, x^2, \frac{1}{x}$ are solutions to the ODE $x^3y''' + x^2y'' - 2xy' + 2y = 0$.

$y = x$, $y' = 1$, $y'' = 0$, $y''' = 0$

$x^3(0) + x^2(0) - 2x(1) + 2x = 0$ true

$y = x^2$, $y' = 2x$, $y'' = 2$, $y''' = 0$

$x^3(0) + 2(x^2) - 2x(2x) + 2x^2 = 0$ true

$y = \frac{1}{x}$, $y' = \frac{-1}{x^2}$, $y'' = \frac{2}{x^3}$, $y''' = \frac{-6}{x^4}$

$x^3\left(-\frac{6}{x^4}\right) + x^2\left(\frac{2}{x^3}\right) - 2x\left(-\frac{1}{x^2}\right) + \frac{2}{x} =$

$-\frac{6}{x} + \frac{2}{x} + \frac{2}{x} + \frac{2}{x} = 0$ true

they are all solutions

4. Solve the IVP $y''' - y'' + y' - y = 0, y(0) = 2, y'(0) = -1, y''(0) = -2$.

$r^3 - r^2 + r - 1 = 0$

$r^2(r-1) + 1(r-1) = 0$

$(r^2+1)(r-1) \quad r = 1, \pm i$

$y(t) = c_1 e^t + c_2 \cos t + c_3 \sin t$

$y(0) = c_1 + c_2 = 2$

$y'(t) = c_1 e^t - c_2 \sin t + c_3 \cos t$

$y'(0) = c_1 + c_3 = -1$

$y''(t) = c_1 e^t - c_2 \cos t - c_3 \sin t$

$y''(0) = c_1 - c_2 = -2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & -1 \\ 1 & -1 & 0 & -2 \end{array}\right]$$

$c_1 = 0$

$c_2 = 2$

$c_3 = -1$

$$\boxed{y(t) = 2\cos t - \sin t}$$

5. Solve the system $\begin{cases} 2x_1 + x_2 - 2x_3 = 3 \\ x_1 - x_2 - x_3 = 0 \\ x_1 + x_2 + 3x_3 = 12 \end{cases}$ using Cramer's Rule. (You can use your calculator to find the necessary determinants.)

$$A_0 = \begin{vmatrix} 2 & 1 & -2 \\ 1 & -1 & -1 \\ 1 & 1 & 3 \end{vmatrix} = -12$$

$$A_1 = \begin{vmatrix} 3 & 1 & -2 \\ 0 & -1 & -1 \\ 12 & 1 & 3 \end{vmatrix} = -42$$

$$A_2 = \begin{vmatrix} 2 & 3 & -2 \\ 1 & 0 & -1 \\ 1 & 12 & 3 \end{vmatrix} = -12$$

$$A_3 = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 1 & 12 \end{vmatrix} = -30$$

$$X_1 = \frac{-42}{-12} = \frac{7}{2}$$

$$X_2 = \frac{-12}{-12} = 1$$

$$X_3 = \frac{-30}{-12} = \frac{5}{2}$$

$$\left(\frac{7}{2}, 1, 5/2\right)$$