

MAT 100 Key Homework # 8

(1)

Lai 460

i. 231

ii. 116 (4 voters $460/4+1$)

iv. $A = 150 \leftarrow A$ wins
 $B = 50$
 $C = 120$
 $D = 140$

A 1st
 B 4th
 C 3rd
 D 2nd

v. $A = 120 \times 1 + 50 \times 2 + 40 \times 3 + 90 \times 4 + 60 \times 4 + 100 \times 2 = 1140 \leftarrow$ Third
 $B = 120 \times 2 + 50 \times 4 + 40 \times 2 + 90 \times 2 + 60 \times 1 + 100 \times 3 = 1060 \leftarrow$ 4th
 $C = 120 \times 4 + 50 \times 3 + 40 \times 1 + 90 \times 3 + 60 \times 2 + 100 \times 1 = 1160 \leftarrow$ 2nd
 $D = 120 \times 3 + 50 \times 1 + 40 \times 4 + 90 \times 1 + 60 \times 3 + 100 \times 4 = 1240 \leftarrow$ D wins

vi. $A \vee B$
 $120 + 50 + 100$

$B \vee C$
 $120 + 90 + 60$

$A \vee C$
 $120 + 50$

$B \vee D$
 $120 + 40 + 60 + 100$

$A \vee D$
 $120 + 40 + 100$

$C \vee D$
 $120 + 50 + 90$

A	B	C	D
	1	111	11
4th	3rd	1st	2nd

C wins

vii. Round 1
 $A: 150$
 ~~$B: 50$~~
 $C: 120$
 $D: 140$

Round 2
 $A: 150$
 $C: 170$
 ~~$D: 140$~~

Round 3
 $A: 290$
 $C: 170$

A wins
 C 2nd B 4th
 D 3rd

viii. Yes, independence of irrelevant alternatives & Condorcet

i. 44
 b. ii. 23

iii. 12 ($44/4+1$)

iv. $A = 18$
 $B = 11$ A wins
 $C = 9$
 $D = 6$

A 1st
 B 2nd
 C 3rd
 D 4th

1b cont'd

V. A: $18 \times 4 + 11 \times 1 + 9 \times 1 + 6 \times 1 = 98$
 B: $18 \times 2 + 11 \times 4 + 9 \times 3 + 6 \times 2 = 119 \leftarrow \text{wins}$
 C: $18 \times 1 + 11 \times 3 + 9 \times 4 + 6 \times 3 = 105$
 D: $18 \times 3 + 11 \times 2 + 9 \times 2 + 6 \times 4 = 118$

B 1st
 D 2nd
 C 3rd
 A 4th

Vi. A v B B v C
 18 18+11
 A v C B v D
 18 18+6
 A v D C v D
 18 18+6

A B C D
 11 1 111
 4th 2nd 3rd ↑
 Darius

vii. Round 1 Round 2 Round 3 A 2nd
 A: 18 A: 18 A: 18 B 3rd
 B: 11 B: 11 C: 26 \leftarrow wins D 4th
 C: 9
 D: ~~6~~ C: 15

viii. yes, independence of irrelevant alternatives & Condorcet

- c. i. 21
- ii. 11
- iii. 5

iv. N: 5
 H: 8 \leftarrow wins
 B: 3
 F: 5
 S: 0

v. N: $5 \times 5 + 3 \times 2 + 3 \times 2 + 5 \times 3$, $3 \times 4 + 2 \times 4 = 72$ winner
 H: $5 \times 2 + 3 \times 5 + 3 \times 3 + 5 \times 5 + 3 \times 1 + 2 \times 2 = 66$
 B: $5 \times 1 + 3 \times 3 + 3 \times 5 + 5 \times 4 + 3 \times 3 + 2 \times 3 = 64$
 F: $5 \times 3 + 3 \times 4 + 3 \times 1 + 5 \times 2 + 3 \times 3 + 2 \times 5 = 46$
 S: $5 \times 4 + 3 \times 1 + 3 \times 4 + 5 \times 1 + 3 \times 2 + 2 \times 1 = 48$

N 1st
 2nd H
 3rd B
 S 4th
 F 5th

12. cont'd

vi

H vs S 5+3+3	S vs N 5+3+3	N vs F 5+3+5	H	B	F	S	N
H vs B 3+3+2	S vs B 5	B vs F 5+3+3	11	111	1	11	
H vs F 5+3+3	S vs F 3+5+3			↑			
H vs N 5+3+2	N vs B 3+3+5			Fwins 1st			

2nd time H, B, N
3rd (test) S

vii. Rnd 1

N: 5
H: 8
B: 3
F: 5
S: 0
↳ 5th

Rnd 2

N: 5
H: 8
~~B: 3~~
F: 5
B 4th

Rnd 3

N: 5
H: 11 ← wins
F: 5

2nd & 3 F tied

viii. Condorcet is violated

2. When they believe their candidate will lose regardless, but they can maybe prevent a less well-liked candidate from winning by voting for a 2nd favorite

3. Answers will vary

4.

# of Votes	1	2	3
1st	A	B	C
2nd	B	C	D
3rd	C	D	B
4th	D	A	A

5. a majority is more than 50% of votes; a plurality is the most votes. These need only be the same when there are only 2 candidates

6. Is there a majority? If there is no majority, there can be no relation.
7. No run-off election is needed (cheaper). ensures someone gets a majority. Less strategic voting
8. Monotonicity requires votes to change (2 elections or a straw poll to compare to is needed) and a candidate needs to get more votes (their position improves) but they end up worse off