142 Homework #4 Key

1.a. $f(x) = 5x - 9 \Rightarrow y = 5x - 9 \Rightarrow x = 5y - 9 \Rightarrow \frac{x+9}{5} = y = f^{-1}(x)$

one-to-one $\quad 5\left(\frac{x+9}{5}\right) - 9 = x + 9 - 9 = x \quad D_{f^{-1}}$: all reals

b. $f(x) = x^3 + 2 \Rightarrow y = x^3 + 2 \Rightarrow x = y^3 + 2 \Rightarrow x - 2 = y^3 \Rightarrow y = \sqrt[3]{x-2} = f^{-1}(x)$

one-to-one

$\left(\sqrt[3]{x-2}\right)^3 + 2 = x - 2 + 2 = x \quad D_f$: all reals

c. $f(x) = \frac{2x+1}{x-3} \Rightarrow y = \frac{2x+1}{x-3} \Rightarrow x = \frac{2y+1}{y-3} \Rightarrow xy - 3x = 2y + 1 \Rightarrow$

one-to-one

$xy - 2y = 3x + 1 \Rightarrow y(x-2) = 3x + 1 \Rightarrow y = \frac{3x+1}{x-2}$

domain f^{-1}: all reals $\neq 2$ $\quad \frac{2\left(\frac{3x+1}{x-2}\right) + 1}{\left(\frac{3x+1}{x-2}\right) - 3} \cdot \frac{x-2}{x-2} = \frac{6x + 2 + x - 2}{3x + 1 - 3x + 6} = \frac{7x}{7} = x$

d. $f(x) = \sqrt[3]{x} + 1 \Rightarrow y = \sqrt[3]{x} + 1 \Rightarrow x = \sqrt[3]{y} + 1 \Rightarrow x - 1 = \sqrt[3]{y} \Rightarrow (x-1)^3 = y$

one-to-one

domain f^{-1}: all reals $\quad y = \sqrt[3]{(x-1)^3} + 1 = x - 1 + 1 = x$

e. $f(x) = e^{2x+1} \Rightarrow y = e^{2x+1} \Rightarrow x = e^{2y+1} \Rightarrow \ln x = 2y + 1 \Rightarrow (\ln x) - 1 = 2y$

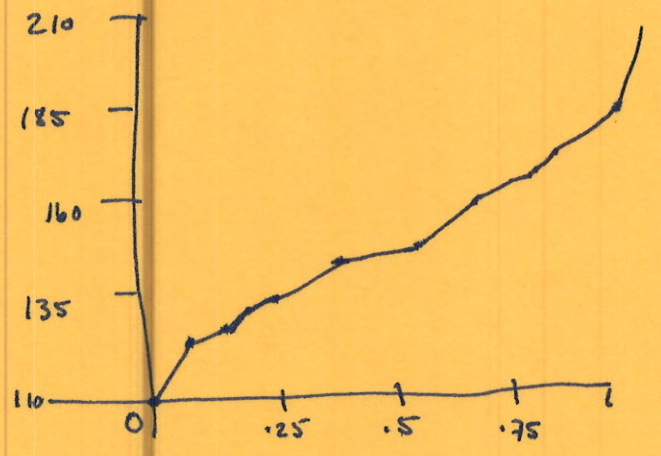
$\Rightarrow \quad y = \frac{1}{2}[\ln x - 1] \quad$ Domain f^{-1}: $(0, \infty)$

one-to-one

$x = e^{2 \cdot \frac{1}{2}(\ln x - 1) + 1} = e^{\ln x - 1 + 1} = e^{\ln x} = x$

f. one-to-one

domain $f^{-1} = [0, 1]$



g. $f(x) = \frac{3}{x-4} \Rightarrow y = \frac{3}{x-4} \Rightarrow x = \frac{3}{y-4}$

one-to-one

$\Rightarrow xy - 4x = 3 \Rightarrow \frac{xy}{x} = \frac{3+4x}{x} \Rightarrow f^{-1} = \frac{3+4x}{x}$

$D_{f^{-1}}$: $x \neq 0$

$\frac{3}{\left(\frac{3+4x}{x}\right) - 4} \cdot \frac{x}{x} = \frac{3x}{3 + 4x - 4x} = \frac{3x}{3} = x$

h. $f(x)=\sqrt{x} \Rightarrow y=\sqrt{x} \Rightarrow x^2=y$ $\quad Df^{-1}: x>0$

one-to-one $\quad \Rightarrow x=\sqrt{y}$

$y=(\sqrt{x})^2=x$

i. $f(x)=x^2-4 \Rightarrow y=x^2-4 \Rightarrow x=y^2-4$ restrict domain to $x>0$

$x+4=y^2 \Rightarrow \sqrt{x+4}=y$ $\quad$ Domain f^{-1}: $x>-4$

$(\sqrt{x+4})^2-4=x+4-4=x$

j. $f(x)=x^2+x+2 \Rightarrow y=x^2+x+\frac{1}{4}-\frac{3}{4}=(x+\frac{1}{2})^2+\frac{7}{4}$ restrict domain to $x>-\frac{1}{2}$

$x=(y+\frac{1}{2})^2+\frac{7}{4} \Rightarrow x-\frac{7}{4}=(y+\frac{1}{2})^2 \Rightarrow$

$\sqrt{(x-\frac{7}{4})}=y+\frac{1}{2} \Rightarrow y=\sqrt{x-\frac{7}{4}}-\frac{1}{2}$ domain $x>\frac{7}{4}$ (f^{-1})

$(\sqrt{x-\frac{7}{4}}-\frac{1}{2})^2+\sqrt{x-\frac{7}{4}}+\frac{1}{2}+2=x-\frac{7}{4}-\sqrt{x-\frac{7}{4}}+\frac{1}{4}+\sqrt{x-\frac{7}{4}}-\frac{1}{2}+2$

$=x-\frac{7}{4}-\frac{1}{4}+2=x-2+2=x$

k. $f(x)=3\ln(x+1) \Rightarrow y=3\ln(x+1) \Rightarrow x=3\ln(y+1) \Rightarrow \frac{x}{3}=\ln(y+1) \Rightarrow$

one-to-one $\quad e^{x/3}=y+1 \Rightarrow e^{x/3}-1=y$ $\quad D$: all reals (f^{-1})

$3\ln(e^{x/3}-1+1)=3\ln(e^{x/3})=3(x/3)=x$

2a. $\frac{3(x+h)+7-(3x+7)}{h}=\frac{3x+3h+7-3x-7}{h}=\frac{3h}{h}=h$

b. $\frac{(x+h)^2-4(x+h)+3-(x^2-4x+3)}{h}=\frac{x^2+2xh+h^2-4x-4h+3-x^2+4x-3}{h}$

$=\frac{h(2x+h-4)}{h}=2x+h-4$

c. $\frac{6-6}{h}=0$

d. $\frac{(\sqrt{x+h-1}-\sqrt{x-1})}{h}\frac{(\sqrt{x+h-1}+\sqrt{x-1})}{(\sqrt{x+h-1}+\sqrt{x-1})}=\frac{x+h-1-(x-1)}{h(\sqrt{x+h-1}+\sqrt{x-1})}=\frac{x+h-1-x+1}{h(\sqrt{x+h-1}+\sqrt{x-1})}=\frac{h}{h(\sqrt{x+h-1}+\sqrt{x-1})}=\frac{1}{\sqrt{x+h-1}+\sqrt{x-1}}$

2e. $\frac{2(x+h)^2 - 2x^2}{h} = \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} = \frac{h(4x+2h)}{h} = 4x+2h$

f. $\frac{-2(x+h)^2 - (-2x^2 - x + 3)}{h} = \frac{-2x^2 - 4xh - 2h^2 - x - h + 3 + 2x^2 + x - 3}{h}$

$= \frac{h(-4x-2h-1)}{h} = -4x-2h-1$

g. $\frac{\frac{1}{2(x+h)} - \frac{1}{2x}}{h} = \frac{2x - (2x+2h)}{h(2x+2h)(2x)} = \frac{2x-2x-2h}{h(2x+2h)(2x)} = \frac{-1}{2x(x+h)}$

3a.
$$\begin{array}{r} x+3 \\ x+5 \overline{)\, x^2+8x+15} \\ -x^2 \mp 5x \\ \hline 3x+15 \\ 3x+15 \\ \hline 0 \end{array}$$

$x+3$

b.
$$\begin{array}{r} 4x^3+16x^2+60x+246 \\ x-4 \overline{)\, 4x^4-4x^2+6x} \\ -4x^4+16x^3 \\ \hline 16x^3-4x^2+6x \\ -16x^3+64x^2 \\ \hline 60x^2+6x \\ -60x^2+240x \\ \hline 246x \\ -246x+984 \end{array}$$

$4x^3+16x^2+60x+246+\frac{984}{x-4}$

c.
$$\begin{array}{r} 6x^2+3x-5 \\ 3x^2+1 \overline{)\, 18x^4+9x^3+3x^2} \\ -18x^4 \quad \mp 18x^2 \\ \hline 9x^3-15x^2 \\ -9x^3 \mp 3x \\ \hline -15x^2-3x \\ +15x^2+5 \\ \hline -3x+5 \end{array}$$

$6x^2+3x-5+\frac{5-3x}{3x^2+1}$

3d.

$$3x+4 \overline{)\,6x^3+17x^2+27x+20}\quad 2x^2+3x+5$$

$-6x^3 \mp 8x^2$

$9x^2+27x$

$-9x^2 \mp 12x$

$15x+20$

$-15x \mp 20$

0

$2x^2+3x+5$

e. $3x^2-x-3 \overline{)\,6x^3+13x^2-11x-15}\quad 2x+5$

$-6x^3+2x^2+6x$

$15x^2-5x-15$

$-15x^2+5x+15$

0

$2x+5$

f. $2x^3+1 \overline{)\,2x^5-8x^4+2x^3+x^2}\quad 2x^2-4x$

$-2x^5 \quad \mp 2x^3$

$-8x^4 \quad +x^2$

$+8x^4 \quad +4x$

x^2+4x

$2x^2-4x+\frac{x^2+4x}{2x^3+1}$

4a. $-3 \overline{)\,5 \;\; -12 \;\; -8}$

$-15 \;\; 81$

$5 \;\; -27 \;\; 73$

$5x-27+\frac{73}{x+3}$

b. $1 \overline{)\,1 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; -2}$

$1 \;\; 1 \;\; 2 \;\; 2 \;\; 2$

$1 \;\; 1 \;\; 2 \;\; 2 \;\; 2 \;\; 0$

$x^4+x^3+2x^2+2x+2$

c. $2\overline{)\,1\ -2\ -1\ \ 3\ -1\ \ 1}$

	2	0	−2	2	2
1	0	−1	1	1	3

$x^4 - x^2 + x + 1 + \frac{3}{x-2}$

d. $2\overline{)\,5\ -6\ \ 3\ \ 11}$

	10	8	22
5	4	11	33

$5x^2 + 4x + 11 + \frac{33}{x-2}$

e. $2\overline{)\,1\ 0\ 0\ 0\ 0\ 0\ 0\ -128}$

	2	4	8	16	32	64	128
1	2	4	8	16	32	64	0

$x^6 + 2x^5 + 4x^4 + 8x^3 + 16x^2 + 32x + 64$