MAT 212, Quiz #6, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Be sure to answer all parts of each question.

1. Use Cramer's Rule to solve the system $\begin{cases} 3x - 4y = 21 \\ 5x + y = 9 \end{cases}$

$A = \begin{bmatrix} 3 & -4 \\ 5 & 1 \end{bmatrix} \Rightarrow \det = 3 + 20 = 23$ $\qquad x_1 = \frac{57}{23}$

$A_1 = \begin{bmatrix} 21 & -4 \\ 9 & 1 \end{bmatrix} \Rightarrow \det = 21 + 36 = 57$ $\qquad x_2 = \frac{-78}{23}$

$A_2 = \begin{bmatrix} 3 & 21 \\ 5 & 9 \end{bmatrix} \Rightarrow \det = 27 - 105 = -78$

2. Solve the second-order ODE for the general solution. [Note: All solutions must be functions of real values only.]

a. $y'' + 6y' + 13y = 0$

$r^2 + 6r + 13 = 0$

$r = \frac{-6 \pm \sqrt{36-52}}{2} = \frac{-6 \pm 4i}{2} = -3 \pm 2i$

$y = c_1 e^{-3t} \cos 2t + c_2 e^{-3t} \sin 2t$

b. $x^2 y'' + xy' + y = 0$

$x^n, \; n x^{n-1}, \; n(n-1) x^{n-2}$

$n^2 - \cancel{n} + \cancel{n} + 1 = 0$

$n^2 + 1 = 0$

$n = \pm i$ $\qquad x^i = e^{i \ln x} \Rightarrow \cos(\ln x)$, $\sin(\ln x)$

$y = c_1 \cos(\ln x) + c_2 \sin(\ln x)$

3. Use the method of undetermined coefficients to solve the particular solution for $35y'' - y' - 12y = e^x$.

$35r^2 - r - 12 = 0$

$(5r+3)(7r-4) = 0$

$r = \frac{-3}{5}, \frac{4}{7}$

$Y = Ae^x$, $Y' = Ae^x$, $Y'' = Ae^x$

$e^x(35A - A - 12A) = e^x$

$e^x(22A) = e^x$

$A = \frac{1}{22}$

$Y(x) = \frac{1}{22}e^x$