

**Instructions:** Show all work. Give exact answers unless specifically asked to round. Be sure to answer all parts of each question.

1. Find the dot product  $\vec{u} \cdot \vec{v}$  for the vectors  $\vec{u} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ ,  $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$ .

$$3(2) + 2(3) + 1(2) = 6 + 6 + 2 = 14$$

2. Solve the general solution for each higher order ODE. For the non-homogeneous case, set up, but do not solve, for the particular solution.

a.  $y''' - y = 0$

$$r^3 - 1 = 0$$

$$(r-1)(r^2+r+1) = 0 \quad r=1$$

$$r = \frac{-1 \pm \sqrt{1-4(1)}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

$$y(t) = c_1 e^t + c_2 e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + c_3 e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)$$

b.  $y^{IV} - 2y'' + y = x \cos x$ .

$$r^4 - 2r^2 + 1 = 0$$

$$(r^2 - 1)^2 = 0$$

$$(r-1)^2(r+1)^2 = 0 \quad r=1, -1$$

$$y(x) = c_1 e^x + c_2 x e^x + c_3 e^{-x} + c_4 x e^{-x}$$

$$Y(x) = Ax \cos x + B \cos x + Cx \sin x + D \sin x$$