MAT 230 Written Homework #5 Key

1. $f(x) = 9x - 3x^2 - x^3$

$x(9 - 3x - x^2) = 0$

$-x(x^2 + 3x - 9) = 0$ $\quad \frac{-3 \pm \sqrt{9+36}}{2} = \frac{-3 \pm \sqrt{45}}{2} = \frac{-3 \pm 3\sqrt{5}}{2} = x$

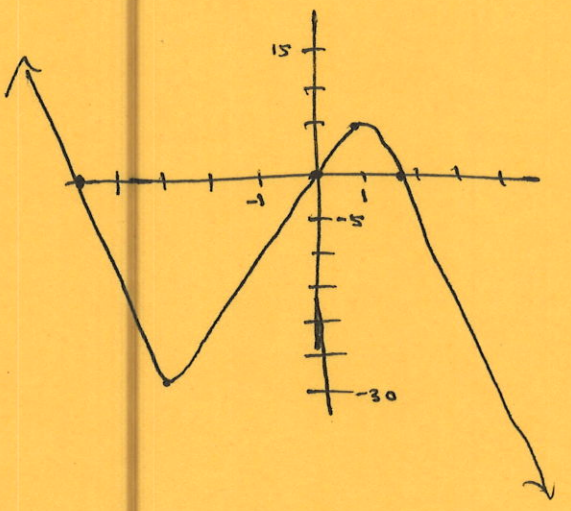
intercepts: $x = 0$, $x = \frac{-3+3\sqrt{5}}{2}$, $\frac{-3-3\sqrt{5}}{2}$

$f'(x) = 9 - 6x - 3x^2$

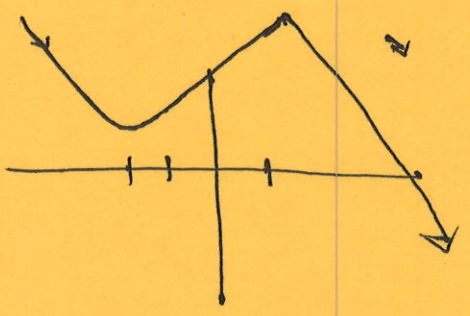
$-3(x^2 + 2x - 3) = 0$

$-3(x+3)(x-1) = 0$ $\quad x = -3, x = 1$

$f(-3) = -27$ $\quad f(1) = 5$

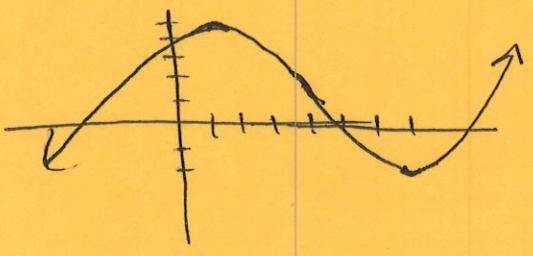


2. a.

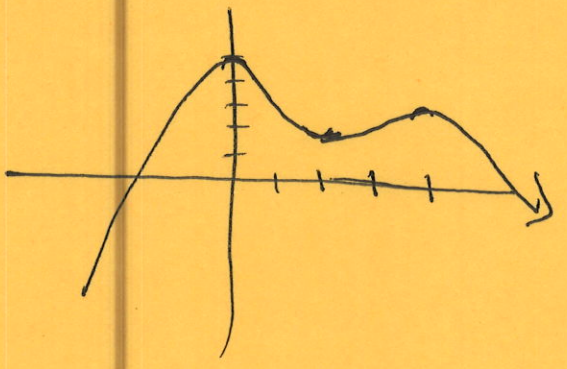


answers will vary

b.

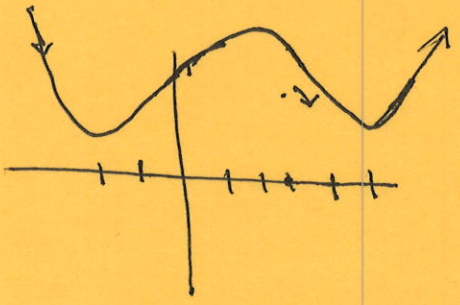


c.

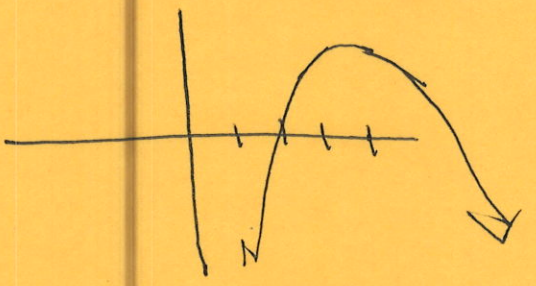


3.

a.



b.

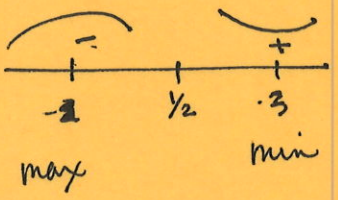


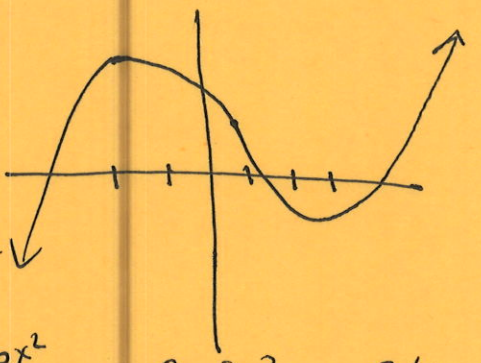
4. a. $f(x) = 2x^3 - 3x^2 - 36x + 28$

$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x-3)(x+2) = 0$

$x = 3, \; x = -2$

$f''(x) = 12x - 6 = 6(2x-1) = 0 \qquad x = 1/2$





b. $f(x) = \dfrac{8x}{x^2+1}$

$f'(x) = \dfrac{8(x^2+1) - 8x(2x)}{(x^2+1)^2} = \dfrac{8x^2 + 8 - 16x^2}{(x^2+1)^2} = \dfrac{8 - 8x^2}{(x^2+1)^2} = \dfrac{8(1-x^2)}{(x^2+1)^2}$

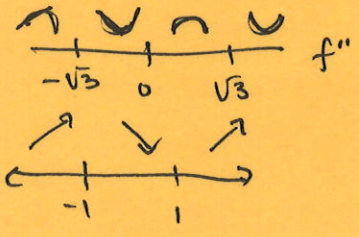
critical points at $1, -1$

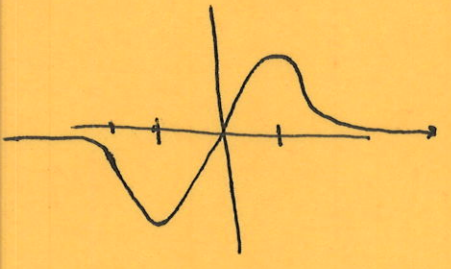
$8(x^2+1)^{-2}(1-x^2)$

$f''(x) = 8(-2)(x^2+1)^{-3}\,2x(1-x^2) + 8(x^2+1)^{-2}(-2x)$

$= \dfrac{-16(2x)(1-x^2) - 16x(x^2+1)}{(x^2+1)^3} = \dfrac{-32x + 32x^3 - 16x^3 - 16x}{(x^2+1)^3} = \dfrac{16x^3 - 48x}{(x^2+1)^3}$

inflection points $\qquad 16x(x^2-3) \qquad$ at $x = 0, \; x = \pm\sqrt{3}$





c. $f(x) = -x\sqrt{1-x^2}$

$f'(x) = -\sqrt{1-x^2} - x(1-x^2)^{-1/2}\left(\tfrac{1}{2}\right)(-2x) = -(1-x^2)^{1/2} + x^2(1-x^2)^{-1/2}$

$= \dfrac{-(1-x^2) + x^2}{(1-x^2)^{1/2}} = \dfrac{-1 + x^2 + x^2}{(1-x^2)^{1/2}} = \dfrac{2x^2 - 1}{(1-x^2)^{1/2}}$

$x = \dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}$ extrema

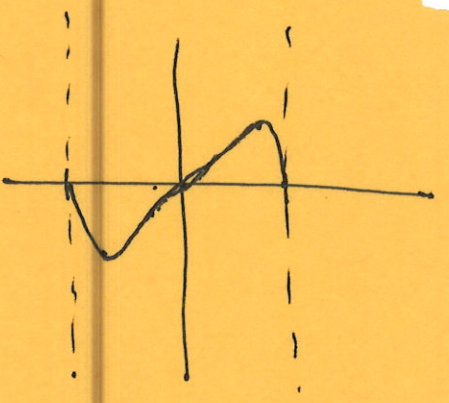
$f''(x) = \dfrac{4x(1-x^2)^{1/2} - \frac{1}{2}(1-x^2)^{-1/2}(-2x)(2x^2-1)}{(1-x^2)} = \dfrac{4x(1-x^2) + 2x^3 - x}{(1-x^2)^{3/2}}$

$\dfrac{4x - 4x^3 + 2x^3 - x}{(1-x^2)^{3/2}} = \dfrac{-2x^3 + 3x}{(1-x^2)^{3/2}}$

4c cont'd

possible inflection points at $x=1, x=-1$ (from denominator)

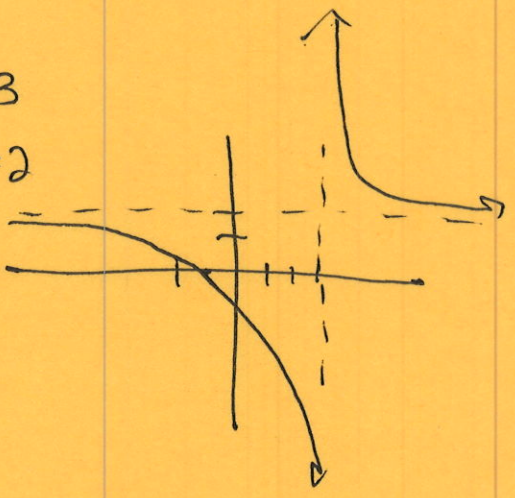
$-x(2x^2-3)=0$ $\quad x=0, \quad x=-\sqrt{\frac{3}{2}}, +\sqrt{\frac{3}{2}}$



5a. $\frac{2x+1}{x-3} = f(x)$

vertical : $x=3$

horizontal $y=2$



$2x+1=0$

$x=-\frac{1}{2}$

b. $f(x) = \frac{x^3}{x^2-1}$

$= x + \frac{x}{x^2-1}$

$$x^2-1 \overline{)\, x^3+0x^2+0x+0} \quad (x)$$
$$-x^3+x$$
$$x$$

oblique/slant $y=x$

vertical at $x=1, x=-1$