MAT230 Homework #6 Key

1. a. $f(x) = x^3 - 3x + 6 \quad [-1, 3]$

$f'(x) = 3x^2 - 3 = 0$

$x^2 - 1 = 0$

$x = \pm 1$

Check: $-1, 1, 3$

$f(1) = 1 - 3 + 6 = 4$ abs min

$f(-1) = -1 + 3 + 6 = 8$

$f(3) = 27 - 9 + 6 = 24$ abs. max.

b. $g(x) = (x+3)^{2/3} - 5 \quad [-4, 5]$

$g'(x) = \frac{2}{3}(x+3)^{-1/3} = 0$ $\quad x = -3$ is undefined

Never

$\frac{2}{3(x+3)^{1/3}}$

check: $-4, -3, 5$

$f(-4) = (-1)^{2/3} - 5 = -4$

$f(-3) = 0^{2/3} - 5 = -5$ abs min

$f(5) = 8^{2/3} - 5 = -1$ abs max

c. $h(x) = \frac{4x}{x^2+1} \quad [-3, 3]$

$$h'(x) = \frac{4(x^2+1) - 2x(4x)}{(x^2+1)^2} = \frac{4x^2 + 4 - 8x^2}{(x^2+1)^2} = \frac{4 - 4x^2}{(x^2+1)^2} = 0$$

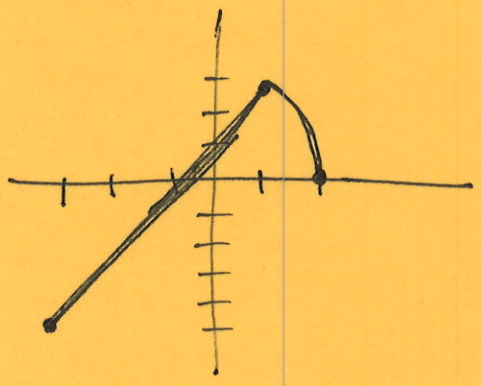
$$4 - 4x^2 = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$$

Check: $-3, -1, 1, 3$

$f(-3) = \frac{-12}{10} = -1.2$ $\qquad f(1) = \frac{4}{2} = 2$ abs max

$f(3) = \frac{12}{10} = 1.2$ $\qquad f(-1) = \frac{-4}{2} = -2$ abs min

2.



$2(-3)+1 = -6+1 = -5$

$2(1)+1 = 3$

$4-(1)^2 = 4-1 = 3$

$4-(2)^2 = 0$

abs min @ 3 = -5

abs max @ 1 = 3

3. $P = 150 - \frac{1}{2}x$ $\quad R(x) = xp = x(150 - \frac{1}{2}x) = 150x - \frac{1}{2}x^2$

$C(x) = 4000 + \frac{1}{4}x$

$P(x) = R(x) - C(x) = 150x - \frac{1}{2}x^2 - (4000 + \frac{1}{4}x) = 149.75x - \frac{1}{2}x^2 - 4000$

$P'(x) = 149.75 - x = 0$

$x = 149.75$ can't produce 149.75 suits so

Check 149 & 150

$P(149) = 7212.25$

$P(150) = 7212.50$ ← slightly higher

150 suits for max profits

$P = 150 - \frac{1}{2}(150) = 75$

4. $2yy' - y - xy' + 2x = 0$

$\frac{(2y-x)y'}{2y-x} = \frac{y-2x}{2y-x} \Rightarrow y' = \frac{y-2x}{2y-x}$