MTH 174 Homework #1 Key

1a. $F'(x) = \frac{1}{x^2}\cdot 2x = \boxed{\frac{2}{x}}$

b. $h'(x) = x^4\cos(x^8)\cdot 4x^3 = \boxed{4x^7\cos(x^8)}$

c. $y'(x) = 2xe^{x^2}\int_0^x e^{-t^2}\,dt + e^{x^2}\left(e^{-x^2}\right) = \boxed{2xe^{x^2}\int_0^x e^{-t^2}\,dt + 1}$

d. $u'(t) = \boxed{e^{\cos(\sin^2 3t)}\cdot\left(-\sin(\sin^2 3t)\right)2\sin 3t\cos 3t\cdot 3}$

e. $g'(t) = \dfrac{\sin(\ln t)\cdot\frac{1}{t}\cdot t - 1\cdot\cos(\ln t)}{t^2} = \boxed{\dfrac{\sin(\ln t)-\cos(\ln t)}{t^2}}$

f. $v'(t) = \sec^2\left(\frac{1}{t}\right)\cdot\frac{-1}{t^2} = \boxed{-\frac{1}{t^2}\sec^2\left(\frac{1}{t}\right)}$

g. $g'(x) = \boxed{-\sqrt{x+\sin x}}$

h. $r'(x) = \dfrac{e^x}{x} - \dfrac{e^{\sqrt{x}}}{\sqrt{x}}\cdot\dfrac{1}{2\sqrt{x}} = \boxed{\dfrac{e^x}{x} - \dfrac{e^{\sqrt{x}}}{2x}}$

i. $a'(t) = \boxed{2^{t-1}(\ln 2)}$

j. $p'(x) = \frac{1}{2}(x^2+3)^{-1/2}\cdot 2x = \boxed{\dfrac{x}{\sqrt{x^2+3}}}$

k. $s'(t) = \dfrac{1}{\sqrt{1-(e^t+t)^2}}\cdot(e^t+1) = \boxed{\dfrac{e^t+1}{\sqrt{1-(e^t+t)^2}}}$

2a. $\int \cos\theta\cos^5(\sin\theta)\,d\theta$ $\qquad u = \sin\theta$, $du = \cos\theta\,d\theta$

$\int \cos^5(u)\,du$

$\int (1-\sin^2 u)^2\cos u\,du$ $\qquad v = \sin u$, $dv = \cos u$

$\int (1-v^2)^2\,dv =$

$\int 1-2v^2+v^4\,dv = v - \frac{2}{3}v^3 + \frac{1}{5}v^5 + C = \sin u - \frac{2}{3}\sin^3 u + \frac{1}{5}\sin^5 v + C$

$= \boxed{\sin(\sin\theta) - \frac{2}{3}\sin^3(\sin\theta) + \frac{1}{5}\sin^5(\sin\theta) + C}$

2b. $\int \dfrac{dx}{\cos x - 1}\cdot\dfrac{\cos x+1}{\cos x+1} = \int \dfrac{\cos x+1}{\cos^2 x-1}\,dx = \int \dfrac{\cos x+1}{-\sin^2 x}\,dx = \int -\cot x\csc x - \csc^2 x\,dx$

$= \boxed{\csc x + \cot x + C}$

2c. $\int \frac{\cos x + \sin 2x}{\sin x}\,dx = \int \frac{\cos x + 2\sin x\cos x}{\sin x}\,dx = \int \cot x + 2\cos x\,dx$ ②

$= \boxed{\ln|\sin x| + 2\sin x + C}$

d. $\int_0^{\pi/4} \sqrt{1-\cos 4\theta}\,d\theta = \int_0^{\pi/4} \sqrt{1-(1-2\sin^2 2\theta)}\,d\theta = \int_0^{\pi/4} \sqrt{2\sin^2 2\theta}\,d\theta$

$= \int_0^{\pi/4} \sqrt{2}\sin 2\theta\,d\theta = \sqrt{2}\cdot\frac{-1}{2}\cos 2\theta\Big|_0^{\pi/4} = -\frac{\sqrt{2}}{2}\cancel{\cos(\pi/2)}^{0} + \frac{\sqrt{2}}{2}\cos(0)$

$= \boxed{\frac{\sqrt{2}}{2}}$

e. $\int \frac{\sin^3\sqrt{x}}{\sqrt{x}}\,dx =$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

$\int 2\sin^3 u\,du$

$2\int (1-\cos^2 u)\sin u\,du$

$v = \cos u$

$dv = -\sin u\,du$

$-2\int 1 - v^2\,dv = -2\left[v - \frac{1}{3}v^3\right] + C = -2\left[\cos u - \frac{1}{3}\cos^3 u\right] + C =$

$\boxed{-2\cos\sqrt{x} + \frac{2}{3}\cos^3\sqrt{x} + C}$

f. $\int \sec^4 q\,dq = \int \sec^2 q\,(1+\tan^2 q)\,dq =$

$u = \tan q$

$du = \sec^2 q\,dq$

$\int 1 + u^2\,du = u + \frac{1}{3}u^3 + C = \boxed{\tan q + \frac{1}{3}\tan^3 q + C}$

g. $\int \frac{1}{\tan x + 1}\,dx = \int \frac{1}{\frac{\sin x}{\cos x} + 1}\,dx \,\frac{\cos x}{\cos x} = \int \frac{\cos x}{\sin x + \cos x}\,dx \cdot \frac{\cos x - \sin x}{\cos x - \sin x} =$

$\int \frac{\cos^2 x - \sin x\cos x}{\cos^2 x - \sin^2 x}\,dx = \int \frac{\frac{1}{2}(1+\cos 2x) - \frac{1}{2}\sin 2x}{\cos 2x}\,dx = \frac{1}{2}\int \sec 2x + 1 - \tan 2x\,dx$

$= \frac{1}{2}\left[\frac{1}{2}\ln|\sec 2x + \tan 2x| + x + 1/2\ln|\cos 2x|\right] + C =$

$\boxed{\frac{1}{4}\ln|\sec 2x + \tan 2x| + \frac{1}{2}x + \frac{1}{4}\ln|\cos 2x| + C}$

3a. $\int \tan^5\varphi \sec^4\varphi \, d\varphi$ $\qquad \sec^2\varphi = 1+\tan^2\varphi$

$\int \tan^5\varphi\,(1+\tan^2\varphi)\sec^2\varphi\,d\varphi$ $\qquad u=\tan\varphi$

$\qquad du = \sec^2\varphi\,d\varphi$

$$\boxed{\int u^5(1+u^2)\,du}$$

(can also be done by converting to $\sec\varphi = u$)

b. $\int \cot^5\varphi \csc^9\varphi\,d\varphi$

$\int(\cot\varphi\csc\varphi)\cot^4\varphi\csc^8\varphi\,d\varphi$ $\qquad \cot^2\varphi = \csc^2\varphi - 1$

$\int(\cot\varphi\csc\varphi)(\csc^2\varphi-1)^2\csc^8\varphi\,d\varphi$ $\qquad u=\csc\varphi$

$\qquad du = -\csc\varphi\cot\varphi\,d\varphi$

$$\boxed{-\int(u^2-1)^2u^8\,du}$$

c. $\int \sin^4\alpha\cos^8 2\alpha\,d\alpha$

$\int\left[\frac{1}{2}(1-\cos 2\alpha)\right]^2\cos^8 2\alpha\,d\alpha$

$\frac{1}{4}\int(1-2\cos 2\alpha+\cos^2 2\alpha)\cos^8 2\alpha\,d\alpha$

$\frac{1}{4}\int \cos^8 2\alpha - 2\cos^9 2\alpha + \cos^{10}2\alpha\,d\alpha$

$\frac{1}{4}\int\left[\frac{1}{2}(1+\cos 4\alpha)\right]^4 - 2(1-\sin^2 2\alpha)^4\cos 2\alpha + \left[\frac{1}{2}(1+\cos 4\alpha)\right]^5 d\alpha$

$\frac{1}{4}\int\frac{1}{16}(1+4\cos 4\alpha+6\cos^2 4\alpha+4\cos^3 4\alpha+\cos^4 4\alpha)+\frac{1}{32}(1+5\cos 4\alpha+$

$\qquad 10\cos^2 4\alpha+10\cos^3 4\alpha+5\cos^4 4\alpha+\cos^5 4\alpha) - 2(1-\sin^2 2\alpha)^4\cos 2\alpha\,d\alpha =$

$\frac{1}{4}\int\frac{3}{32}+\frac{13}{32}\cos 4\alpha+\frac{11}{16}\cos^2 4\alpha+\frac{9}{16}\cos^3 4\alpha+\frac{7}{32}\cos^4 4\alpha+\frac{1}{32}\cos^5 4\alpha$

$\qquad -2(1-\sin^2 2\alpha)^4\cos 2\alpha\,d\alpha$

$\frac{1}{4}\int\frac{3}{32}+\frac{13}{32}\cos 4\alpha+\frac{9}{16}(1-\sin^2 4\alpha)\cos 4\alpha+\frac{1}{32}(1-\sin^2 4\alpha)^2\cos 4\alpha-2(1-\sin^2 2\alpha)^4\cos 2\alpha$

$\qquad +\frac{11}{16}\left[\frac{1}{2}(1+\cos 8\alpha)\right]+\frac{7}{32}\left[\frac{1}{2}(1+\cos 8\alpha)\right]^2 d\alpha =$

$\frac{1}{4}\int\frac{3}{32}+\frac{13}{32}\cos 4\alpha+\frac{9}{16}(1-\sin^2 4\alpha)\cos 4\alpha+\frac{1}{32}(1-\sin^2 4\alpha)^2\cos 4\alpha-2(1-\sin^2 2\alpha)^4\cos 2\alpha$

$\qquad +\frac{11}{32}+\frac{11}{32}\cos 8\alpha+\frac{7}{32}\left(\frac{1}{4}\right)(1+2\cos 8\alpha+\cos^2 8\alpha)\;d\alpha =$

3c. cont'd

$$\frac{1}{4}\int \frac{3}{32} + \frac{13}{32}\cos 4\alpha + \frac{9}{16}(1-\sin^2 4\alpha)\cos 4\alpha + \frac{1}{32}(1-\sin^2 4\alpha)^2\cos 4\alpha$$

$$- 2(1-\sin^2 2\alpha)^4\cos 2\alpha + \frac{11}{32} + \frac{11}{32}\cos 8\alpha + \frac{7}{128} + \frac{7}{64}\cos 8\alpha + \frac{7}{128}\cos^2 8\alpha \, d\alpha$$

$$= \frac{1}{4}\int \frac{63}{128} + \frac{13}{32}\cos 4\alpha + \frac{9}{16}(1-\sin^2 4\alpha)\cos 4\alpha + \frac{1}{32}(1-\sin^2 4\alpha)^2\cos 4\alpha$$

$$-2(1-\sin^2 2\alpha)^4\cos 2\alpha + \frac{29}{64}\cos 8\alpha + \frac{7}{128}(1+\cos 16\alpha)\cdot\frac{1}{2}\, d\alpha$$

$$= \frac{1}{4}\int \frac{63}{128} + \frac{13}{32}\cos 4\alpha + \frac{9}{16}(1-\sin^2 4\alpha)\cos 4\alpha + \frac{1}{32}(1-\sin^2 4\alpha)^2\cos 4\alpha$$

$$-2(1-\sin^2 2\alpha)^4\cos 2\alpha + \frac{29}{64}\cos 8\alpha + \frac{7}{256} + \frac{7}{256}\cos 16\alpha\, d\alpha$$

$$= \frac{1}{4}\int \frac{133}{256} + \frac{13}{32}\cos 4\alpha + \frac{29}{64}\cos 8\alpha + \frac{7}{256}\cos 16\alpha\, d\alpha +$$

$$\frac{1}{4}\int \frac{9}{16}(1-\sin^2 4\alpha)\cos 4\alpha + \frac{1}{32}(1-\sin^2 4\alpha)^2\cos 4\alpha\, d\alpha +$$

$$\frac{1}{4}\int -2(1-\sin^2 2\alpha)^4\cos 2\alpha\, d\alpha =$$

$u = \sin 4\alpha$
$du = 4\cos 4\alpha\, d\alpha$
$\frac{1}{4}du = \cos 4\alpha\, d\alpha$

$$= \frac{1}{4}\int \frac{133}{256} + \frac{13}{32}\cos 4\alpha + \frac{29}{64}\cos 8\alpha + \frac{7}{256}\cos 16\alpha\, d\alpha +$$

$v = \sin 2\alpha$
$dv = 2\cos 2\alpha\, d\alpha$
$\frac{1}{2}dv = \cos 2\alpha\, d\alpha$

$$\frac{1}{4}\int \frac{9}{16}(1-u^2)\frac{1}{4}du + \frac{1}{32}(1-u^2)^2\frac{1}{4}du +$$

$$\frac{1}{4}\int -\cancel{2}(1-v^2)^4\left(\frac{1}{\cancel{2}}\right)dv$$

$$=\boxed{\frac{1}{4}\int \frac{133}{256} + \frac{13}{32}\cos 4\alpha + \frac{29}{64}\cos 8\alpha + \frac{7}{256}\cos 16\alpha\, d\alpha + \frac{9}{256}\int(1-u^2)\,du + \frac{1}{512}\int(1-u^2)^2\,du - \frac{1}{4}\int(1-v^2)^4\,dv}$$

3d. $\int \cos^{10}\beta\, d\beta = \int \left[\frac{1}{2}(1+\cos 2\beta)\right]^5 d\beta =$

$$\frac{1}{32}\int 1 + 5\cos 2\beta + 10\cos^2 2\beta + 10\cos^3 2\beta + 5\cos^4 2\beta + \cos^5 2\beta\, d\beta$$

$$\frac{1}{32}\int 1 + 5\cos 2\beta + 10(1-\sin^2 2\beta)\cos 2\beta + (1-\sin^2 2\beta)^2\cos 2\beta + 10\cdot\frac{1}{2}(1+\cos 4\beta) +$$

$$5\left[\frac{1}{2}(1+\cos 4\beta)\right]^2 d\beta =$$

$$\frac{1}{32}\int 1 + 5\cos 2\beta + 10(1-\sin^2 2\beta)\cos 2\beta + (1-\sin^2 2\beta)^2\cos 2\beta + 5(1+\cos 4\beta) +$$

$$\frac{5}{4}(1 + 2\cos 4\beta + \cos^2 4\beta)\, d\beta =$$

3d cont'd

$$\frac{1}{32}\int \frac{29}{4} + 5\cos 2\beta + 10(1-\sin^2 2\beta)\cos 2\beta + (1-\sin^2 2\beta)^2\cos 2\beta + 5\cos 4\beta$$
$$+\frac{5}{2}\cos 4\beta + \frac{5}{4}\cdot\frac{1}{2}(1+\cos 8\beta)\, d\beta =$$

$$\frac{1}{32}\int \frac{63}{8} + 5\cos 2\beta + 10(1-\sin^2 2\beta)\cos 2\beta + (1-\sin^2 2\beta)^2\cos 2\beta + \frac{15}{2}\cos 4\beta + \frac{5}{8}\cos 8\beta \, d\beta$$

$$= \frac{1}{32}\int \frac{63}{8} + 5\cos 2\beta + \frac{15}{2}\cos 4\beta + \frac{5}{8}\cos 8\beta \, d\beta +$$
$$+\frac{1}{32}\int 5(1-u^2)\,du + \frac{1}{2}(1-u^2)^2\,du$$

$u = \sin 2\beta$
$du = 2\cos 2\beta \, d\beta$
$\frac{1}{2}du = \cos 2\beta \, d\beta$

$$= \boxed{\frac{1}{32}\int \frac{63}{8} + 5\cos 2\beta + \frac{15}{2}\cos 4\beta + \frac{5}{8}\cos 8\beta \, d\beta + \frac{1}{32}\int 5(1-u^2) + \frac{1}{2}(1-u^2)^2\, du}$$

3e. $\int \cos^{17}\theta \sin^6\theta \, d\theta = \int \cos\theta (1-\sin^2\theta)^8 \sin^6\theta \, d\theta$

$$= \boxed{\int (1-u^2)^8 u^6 \, du}$$

$u = \sin\theta$
$du = \cos\theta \, d\theta$

3f. $\int \sin^6 2\psi \cos^{12} 2\psi \, d\psi = \int \left[\frac{1}{2}(1-\cos 2\psi)\right]^3 \left[\frac{1}{2}(1+\cos 2\psi)\right]^6 d\psi$

$$\frac{1}{512}\int (1-\cos 2\psi)^3(1+\cos 2\psi)^3(1+\cos 2\psi)^3\, d\psi = \frac{1}{512}\int (1-\cos^2 2\psi)^3(1+\cos 2\psi)^3\, d\psi$$

$$\frac{1}{512}\int (1-3\cos^2 2\psi + 3\cos^4 2\psi - \cos^6 2\psi)(1+3\cos 2\psi + 3\cos^2 2\psi + \cos^3 2\psi)\, d\psi$$

$$= \frac{1}{512}\int 1 + 3\cos 2\psi + \cancel{3\cos^2 2\psi} + \cos^3 2\psi - \cancel{3\cos^2 2\psi} - 9\cos^3 2\psi - 9\cos^4 2\psi - 3\cos^5 2\psi$$
$$+ 3\cos^4 2\psi + 9\cos^5 2\psi + 9\cos^6 2\psi + \cancel{3\cos^7 2\psi} - \cos^6 2\psi - \cancel{3\cos^7 2\psi}$$
$$- 3\cos^8 2\psi - \cos^9 2\psi \, d\psi$$

$$= \frac{1}{512}\int 1 + 3\cos 2\psi - 8\cos^3 2\psi - 6\cos^4 2\psi + 6\cos^5 2\psi + 8\cos^6 2\psi - 3\cos^8 2\psi$$
$$- \cos^9 2\psi \, d\psi =$$

$$\frac{1}{512}\int 1 + 3\cos 2\psi - 8(1-\sin^2 2\psi)\cos 2\psi + 6(1-\sin^2 2\psi)^2\cos 2\psi - (1-\sin^2 2\psi)^4\cos 2\psi$$
$$- 6\left[\frac{1}{2}(1+\cos 4\psi)\right]^2 + 8\left[\frac{1}{2}(1+\cos 4\psi)\right]^3 - 3\left[\frac{1}{2}(1+\cos 4\psi)\right]^4 d\psi$$

$$\frac{1}{512}\int 1+3\cos 2\psi-8(1-\sin^2 2\psi)\cos 2\psi+6(1-\sin^2 2\psi)^2\cos 2\psi-(1-\sin^2 2\psi)^4\cos 2\psi$$

$$-\frac{6}{4}(1+2\cos 4\psi+\cos^2 4\psi)+(1+3\cos 4\psi+3\cos^2 4\psi+\cos^3 4\psi)$$

$$-\frac{3}{16}(1+4\cos 4\psi+6\cos^2 4\psi+4\cos^3 4\psi+\cos^4 4\psi)\,d\psi$$

$$=\frac{1}{512}\int \frac{5}{16}+3\cos 2\psi-8(1-\sin^2 2\psi)\cos 2\psi+6(1-\sin^2 2\psi)^2\cos 2\psi-(1-\sin^2 2\psi)^4\cos 2\psi$$

$$-\frac{3}{4}\cos 4\psi+\frac{3}{8}\cos^2 4\psi+\frac{1}{4}\cos^3 4\psi-\frac{3}{16}\cos^4 4\psi\,d\psi$$

$$=\frac{1}{512}\int \frac{5}{16}+3\cos 2\psi-\frac{3}{4}\cos 4\psi-8(1-\sin^2 2\psi)\cos 2\psi+6(1-\sin^2 2\psi)^2\cos 2\psi$$

$$-(1-\sin^2 2\psi)^4\cos 2\psi+\frac{1}{4}(1-\sin^2 4\psi)\cos 4\psi+\frac{3}{8}\cdot\frac{1}{2}(1+\cos 8\psi)$$

$$-\frac{3}{16}\left[\frac{1}{2}(1+\cos 8\psi)\right]^2 d\psi$$

$$=\frac{1}{512}\int \frac{1}{2}+3\cos 2\psi-\frac{3}{4}\cos 4\psi+\frac{3}{16}\cos 8\psi-8(1-\sin^2 2\psi)\cos 2\psi+6(1-\sin^2 2\psi)^2\cos 2\psi$$

$$-(1-\sin^2 2\psi)^4\cos 2\psi+\frac{1}{4}(1-\sin^2 4\psi)\cos 4\psi-\frac{3}{128}(1+2\cos 8\psi+\cos^2 8\psi)\,d\psi$$

$$=\frac{1}{512}\int \frac{61}{128}+3\cos 2\psi-\frac{3}{4}\cos 4\psi+\frac{9}{64}\cos 8\psi-8(1-\sin^2 2\psi)\cos 2\psi+$$

$$6(1-\sin^2 2\psi)^2\cos 2\psi-(1-\sin^2 2\psi)^4\cos 2\psi+\frac{1}{4}(1-\sin^2 4\psi)\cos 4\psi$$

$$-\frac{3}{128}\cdot\frac{1}{2}(1+\cos 16\psi)\,d\psi$$

$$=\frac{1}{512}\int \frac{119}{256}+3\cos 2\psi-\frac{3}{4}\cos 4\psi+\frac{9}{64}\cos 8\psi-\frac{3}{256}\cos 16\psi\,d\psi+$$

$$\frac{1}{512}\int -8(1-\sin^2 2\psi)\cos 2\psi+6(1-\sin^2 2\psi)^2\cos 2\psi-(1-\sin^2 2\psi)^4\cos 2\psi\,d\psi$$

$$+\frac{1}{512}\int \frac{1}{4}(1-\sin^2 4\psi)\cos 4\psi\,d\psi$$

$u=\sin 2\psi$ $\quad$ $v=\sin 4\psi$

$du=2\cos 2\psi\,d\psi$ $\quad$ $dv=4\cos 4\psi\,d\psi$

$\frac{1}{2}du=\cos 2\psi\,d\psi$ $\quad$ $\frac{1}{4}dv=\cos 4\psi\,d\psi$

$$=\frac{1}{512}\int \frac{119}{256}+3\cos 2\psi-\frac{3}{4}\cos 4\psi+\frac{9}{16}\cos 8\psi-\frac{3}{256}\cos 16\psi\,d\psi$$

$$+\frac{1}{1024}\int -8(1-u^2)+6(1-u^2)^2-(1-u^2)^4\,du+\frac{1}{2048}\int\frac{1}{4}(1-v^2)\,dv$$

3g. $\int \cos^{11}\alpha \sin^3 4\alpha \, d\alpha = \int \cos^{11}\alpha \, (2\sin 2\alpha \cos 2\alpha)^3 \, d\alpha$

$= 8\int \cos^{11}\alpha \sin^3 2\alpha \cos^3 2\alpha \, d\alpha = 8\int \cos^{11}\alpha \, (2\sin\alpha\cos\alpha)^3 (\cos^2\alpha - \sin^2\alpha)^3 \, d\alpha$

$= 64\int \cos^{11}\alpha \sin^3\alpha \cos^3\alpha \, (\cos^6\alpha - 3\cos^4\alpha\sin^2\alpha + 3\cos^2\alpha\sin^4\alpha - \sin^6\alpha) \, d\alpha$

$= 64\int \cos^{14}\alpha \sin^3\alpha \, (\cos^6\alpha - 3\cos^4\alpha\sin^2\alpha + 3\cos^2\alpha\sin^4\alpha - \sin^6\alpha) \, d\alpha$

$= 64\int \cos^{20}\alpha\sin^3\alpha - 3\cos^{18}\alpha\sin^5\alpha + 3\cos^{16}\alpha\sin^7\alpha - \cos^{14}\alpha\sin^9\alpha \, d\alpha$

$= 64\int \cos^{20}\alpha(1-\cos^2\alpha)\sin\alpha - 3\cos^{18}\alpha(1-\cos^2\alpha)^2\sin\alpha + 3\cos^{16}\alpha(1-\cos^2\alpha)^3\sin\alpha$

$- \cos^{14}\alpha(1-\cos^2\alpha)^4\sin\alpha \, d\alpha$

$= 64\int u^{20}(1-u^2)(-du) - 3u^{18}(1-u^2)^2(-du) +$
$3u^{16}(1-u^2)^3(-du) - u^{14}(1-u^2)^4(-du)$

$u = \cos\alpha$
$du = -\sin\alpha \, d\alpha$

$= \boxed{64\int u^{14}(1-u^2)^4 - 3u^{16}(1-u^2)^3 + 3u^{18}(1-u^2)^2 - u^{20}(1-u^2)\, du}$

4a. $\int \frac{x(x-2)}{(x-1)^3} dx$

$u = x-1 \qquad x = u+1$
$du = dx \qquad u-1 = x-2$

$= \int \frac{(u+1)(u-1)}{u^3} du = \int \frac{u^2-1}{u^3} du = \int \frac{1}{u} - \frac{1}{u^3} du = \ln u - \frac{u^{-2}}{-2} + C$

$\ln u + \frac{1}{u^2} + C = \boxed{\ln|x-1| + \frac{1}{(x-1)^2} + C}$

b. $\int \frac{1}{x\ln(x^3)} dx = \int \frac{1}{3x\ln x} dx = \frac{1}{3}\int \frac{1}{u} du$

$u = \ln x$
$du = \frac{1}{x} dx$

$= \frac{1}{3}\ln u + C = \boxed{\frac{1}{3}\ln|\ln|x|| + C}$

c. $\int \frac{1}{(x-1)\sqrt{x^2-2x}} dx = \int \frac{1}{(x-1)\sqrt{(x^2-2x+1)-1}} dx = \int \frac{1}{(x-1)\sqrt{(x-1)^2-1}} dx$

$= \int \frac{du}{u\sqrt{u^2-1}} = \operatorname{arcsec} u + C = \boxed{\operatorname{arcsec}|x-1| + C}$

$u = x-1$
$du = dx$

4d. $\int \frac{\sinh x}{1+\sinh^2 x}\,dx = \int \frac{\sinh x}{\cosh^2 x}\,dx = \int \tanh x\,\operatorname{sech} x\,dx = \boxed{-\operatorname{sech} x + C}$

e. $\int \tan x \ln(\cos x)\,dx$ $\quad u = \ln(\cos x)$, $du = -\tan x\,dx$

$= -\int u\,du = -\frac{1}{2}u^2 + C$

$= \boxed{-\frac{1}{2}\ln^2(\cos x) + C}$

f. $\int \frac{1}{3t+1} - \frac{17}{(4t-1)^2}\,dt = \boxed{\frac{1}{3}\ln|3t+1| + \frac{17}{4(4t-1)} + C}$

g. $\int \frac{\sec x \tan x}{\sec x - 1}\,dx$

$u = \sec x - 1$
$du = \sec x \tan x\,dx$

$\int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|\sec x - 1| + C}$

h. $\int_{-2}^{2} \frac{dx}{x^2+4x+13} = \int_{-2}^{2} \frac{dx}{(x^2+4x+4)+9} = \int_{-2}^{2} \frac{dx}{(x+2)^2+9}$ $\quad u = x+2$, $du = dx$

$\int_0^4 \frac{du}{u^2+9} = \frac{1}{3}\arctan\left(\frac{u}{3}\right)\Big|_0^4 = \boxed{\frac{1}{3}\arctan\left(\frac{4}{3}\right)}$

i. $\int \frac{\arccos x}{\sqrt{1-x^2}}\,dx$

$u = \arccos x$
$du = \frac{-1}{\sqrt{1-x^2}}\,dx$

$= -\int u\,du = -\frac{1}{2}u^2 + C = \boxed{-\frac{1}{2}\arccos^2 x + C}$

j. $\int \frac{5}{3e^x - 2} \cdot \frac{e^{-x}}{e^{-x}}\,dx = \int \frac{5e^{-x}}{3-2e^{-x}}\,dx$

$u = 3-2e^{-x}$
$du = 2e^{-x}\,dx$

$\frac{5}{2}\int \frac{1}{u}\,du = \frac{5}{2}\ln u + C = \boxed{\frac{5}{2}\ln|3-2e^{-x}| + C}$

k. $\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}}\,dx = \int \frac{4}{4(x-1)\sqrt{4(x^2-2x+1)-1}}\,dx = \int \frac{2}{2(x-1)\sqrt{4(x-1)^2-1}}\,dx$

$= \int \frac{du}{u\sqrt{u^2-1}} = \operatorname{arcsec} u + C = \boxed{\operatorname{arcsec}[2(x-1)] + C}$

$u = 2(x-1)$
$du = 2\,dx$

4l. $\int 3^t\,dt = \boxed{\frac{3^t}{\ln 3} + C}$

5. $\frac{1}{4}\left[0 - 2(.7) + 0(2) + 2(.4) + 2(.6) + 2(.8) + 2(.9) + 2(.95) + 2(.99) + 2(1) + 1\right]$

≈ 2.72

6. $\frac{10}{3}\big[5000 + 4(5000) + 2(9000) + 4(15000) + 2(14000) + 4(9000) + 2(17000) +$
$4(28000) + 2(40{,}000) + 4(46{,}000) + 2(60{,}000) + 4(80{,}000) + 2(85{,}000)$
$+ 4(90{,}000) + 2(70{,}000) + 4(95{,}000) + 2(105{,}000) + 4(96{,}000) + 2(94{,}000)$
$+ 4(88{,}000) + 2(90{,}000) + 4(98{,}000) + 2(105{,}000) + 4(120{,}000)$
$+ 130{,}000\big] = 15{,}296{,}667$

≈ 15.3 million

7a. i. $\frac{1}{2}\left[2\sqrt[3]{1} + 2\sqrt[3]{2} + 2\sqrt[3]{3} + 2\sqrt[3]{4} + 2\sqrt[3]{5} + 2\sqrt[3]{6} + 2\sqrt[3]{7} + \sqrt[3]{8}\right] \approx 11.729599\ldots$

ii. $\frac{1}{4}\left[\sqrt{\ln 1} + 2\sqrt{\ln 1.5} + 2\sqrt{\ln 2} + 2\sqrt{\ln 2.5} + 2\sqrt{\ln 3} + 2\sqrt{\ln 3.5} + \sqrt{\ln 4}\right] \approx 2.516797\ldots$

iii. $\frac{1}{10}\big[e^{e^{-1}} + 2e^{e^{-.8}} + 2e^{e^{-.6}} + 2e^{e^{-.4}} + 2e^{e^{-.2}} + 2e^{e^{0}} + 2e^{e^{.2}} + 2e^{e^{.4}} + 2e^{e^{.6}} +$
$2e^{e^{.8}} + e^{e^{1}}\big] \approx 8.36385\ldots$

b. i. $\frac{1}{3}\left[0 + 4\sqrt[3]{1} + 2\sqrt[3]{2} + 4\sqrt[3]{3} + 2\sqrt[3]{4} + 4\sqrt[3]{5} + 2\sqrt[3]{6} + 4\sqrt[3]{7} + \sqrt[3]{8}\right] \approx 11.863\ldots$

ii. $\frac{1}{6}\left[\sqrt{\ln 1} + 4\sqrt{\ln 1.5} + 2\sqrt{\ln 2} + 4\sqrt{\ln 2.5} + 2\sqrt{\ln 3} + 4\sqrt{\ln 3.5} + \sqrt{\ln 4}\right] \approx 2.631976\ldots$

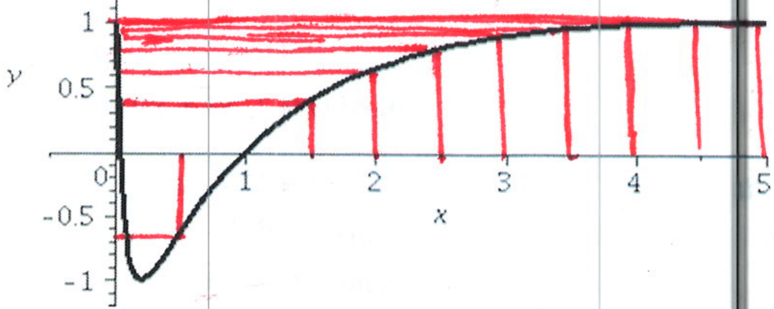
iii. $\frac{1}{15}\big[e^{e^{-1}} + 4e^{e^{-.8}} + 2e^{e^{-.6}} + 4e^{e^{-.4}} + 2e^{e^{-.2}} + 4e^{e^{0}} + 2e^{e^{.2}} + 4e^{e^{.4}} + 2e^{e^{.6}} + 4e^{e^{.8}}$
$+ e^{e^{1}}\big] \approx 8.235\ldots$

c. i. $= 12$

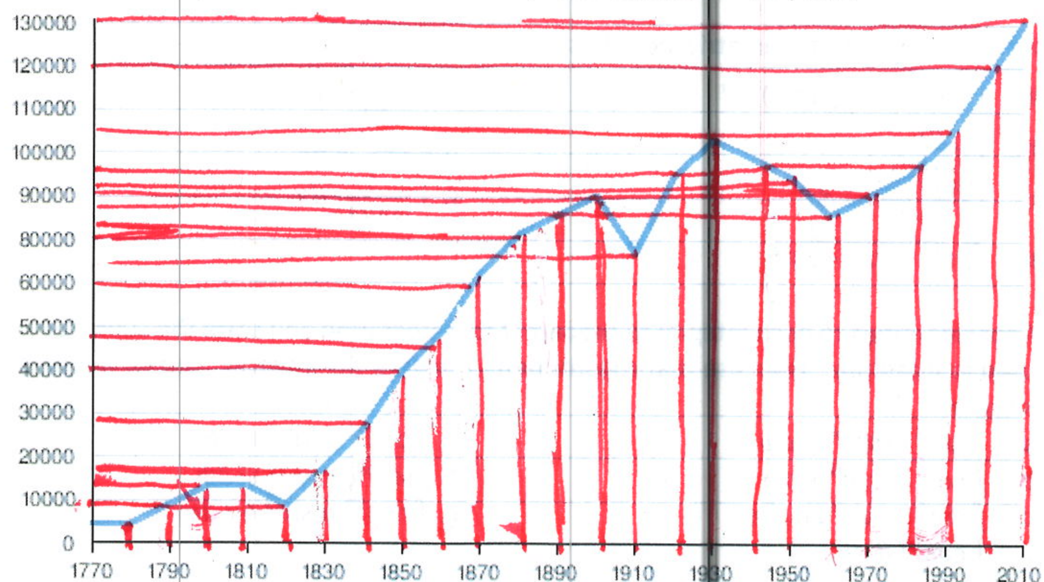
f. $\int \frac{1}{3t+1} - \frac{17}{(4t-1)^2} dt$

l. $\int 3^t dt$

5. Using the graph below and the Trapezoidal Rule, estimate the area bounded by the graph and the x-axis, with $\Delta x = 0.5$ between 0 and 5.



6. Using the graph below and Simpson's Rule, estimate the area under the graph between 1770 and 2010. Use units of ten-thousands for the y-values and $\Delta t = 10$ years.



7. Numerically integrate each of the following functions using:
   a. The Trapezoidal Rule
   b. Simpson's Rule
   c. Compare your results to the true value from the Fundamental Theorem of Calculus (for (i) only).

   i. $\int_0^8 \sqrt[3]{x}dx$, $n = 8$

   ii. $\int_1^4 \sqrt{\ln x}\, dx$, $n = 6$

   iii. $\int_{-1}^{1} e^{e^x} dx$, $n = 10$

8. Use the Error formulas to calculate the number of partitions needs to calculate each integral to within 0.000001 for:
   a. The Trapezoidal Rule

8. $10^{-6} = E$

Trapezoidal: $E \le \dfrac{K(b-a)^3}{12n^2}$ $\quad K = \max|f''|$

Simpson's: $E \le \dfrac{M(b-a)^5}{180n^4}$ $\quad M = \max|f^{iv}(x)|$

a. $10^{-6} =$

i. $f(x) = x^{-1/2}$

$f'(x) = -\frac{1}{2}x^{-3/2}$

$f''(x) = \frac{3}{4}x^{-5/2}$

$f'''(x) = -\frac{15}{8}x^{-7/2}$

$f^{iv}(x) = +\frac{105}{16}x^{-9/2}$

$10^{-6} = \dfrac{\frac{3}{4}(3-1)^3}{12n^2} \Rightarrow n^2 = \dfrac{\frac{3}{4}(8)}{12}\cdot 10^6$

$\boxed{n = 708}$

$\left|\frac{3}{4}\frac{1}{1^{5/2}}\right| = \frac{3}{4}$ $\quad \frac{3}{4}\left|\frac{1}{3^{5/2}}\right| < \frac{3}{4}$

bi. $\max|f^{iv}(x)| = \frac{105}{16}$

$10^{-6} = \dfrac{\frac{105}{16}(3-1)^5}{180n^4} \Rightarrow n^4 = \dfrac{\frac{105}{16}(32)}{180}10^6$

$\boxed{n = 34}$

a.ii. $f(x) = (4-x^3)^{1/2}$

$f'(x) = -\frac{1}{2}(4-x^3)^{-1/2}\cdot 3x^2 = -\frac{3}{2}x^2(4-x^3)^{-1/2}$

$f''(x) = -3x(4-x^3)^{-1/2} + -\frac{3}{2}(-1/2)x^2(4-x^3)^{-3/2}(-3x^2) \rightarrow$ max $\quad$ (1) (−1) $\boxed{1.3}$ vs -1.14

$= -3x(4-x^3)^{-1/2} - \frac{9}{4}x^4(4-x^3)^{-3/2}$

$f'''(x) = -3(4-x^3)^{-1/2} - \frac{9}{4}(4x^3)(4-x^3)^{-3/2} - \frac{9}{4}x^4(4-x^3)^{-5/2}\left(-\frac{3}{2}\right)(-3x^2)$

$-3x(4-x^3)^{-3/2}\left(-\frac{1}{2}\right)(-3x^2)$

$= -3(4-x^3)^{-1/2} - \frac{27}{2}x^3(4-x^3)^{-3/2} - \frac{81}{8}x^6(4-x^3)^{-5/2}$

$f^{iv}(x) = -3(-1/2)(4-x^3)^{-3/2}(-3x^2) - \frac{27}{2}(3x^2)(4-x^3)^{-3/2} - \frac{27}{2}x^3(4-x^3)^{-5/2}\left(-\frac{3}{2}\right)(-3x^2)$

$-\frac{81}{8}(6x^5)(4-x^3)^{-5/2} - \frac{81}{8}x^6(4-x^3)^{-7/2}\left(-\frac{5}{2}\right)(-3x^2)$

$= -45(4-x^3)^{-3/2} - \left(\frac{81}{2}x^2 + \frac{243}{4}x^5\right)(4-x^3)^{-5/2} - \left(\frac{243}{4}x^5 + \frac{1215}{16}x^8\right)(4-x^3)^{-7/2}$

max → (1) -18.08 vs (−1) -3.717

$10^{-6} = \dfrac{1.3(2)^3}{12n^2}$

$n^2 = \dfrac{1.3\cdot 8}{12}10^6$

$\boxed{n = 931}$

bii. $10^{-6} = \dfrac{18.08(2)^5}{180n^4}$

$n^4 = \dfrac{18.08(32)}{180}10^6$

$\boxed{n = 44}$

8a. iii. $f(x) = e^{1/x}$ (11)

$f'(x) = e^{1/x} \cdot (-x^{-2})$

$f''(x)\ e^{1/x}(-x^{-2})(-x^{-2}) + e^{-1/x}(2x^{-3}) = e^{1/x}(x^{-4} + 2x^{-3})$ $\quad e^1(1+2) = 3e^1$

$f'''(x) = e^{1/x}(-x^{-2})(x^{-4}+2x^{-3}) + e^{1/x}(-4x^{-5} - 6x^{-4})$

$= e^{1/x}(-x^{-6} - 2x^{-5} - 4x^{-5} - 6x^{-4}) = e^{1/x}(-x^{-6} - 6x^{-5} - 6x^{-4})$

$f^{iv}(x) = e^{1/x}(-x^{-2})(-x^{-6} - 6x^{-5} - 6x^{-4}) + e^{1/x}(6x^{-7} + 30x^{-6} + 24x^{-5})$

$= e^{1/x}(x^{-8} + 6x^{-7} + 6x^{-6} + 6x^{-7} + 30x^{-6} + 24x^{-5})$

$= e^{1/x}(x^{-8} + 12x^{-7} + 36x^{-6} + 24x^{-5})$

max at 1

$e^1(1+12+36+24) = 73e$

$10^{-6} = \dfrac{3e(1)^3}{12n^2} \Rightarrow n^2 = \dfrac{3e}{12} 10^6$

$\boxed{n = 825}$

b iii: $10^{-6} = \dfrac{73e(1)^5}{180n^4} \Rightarrow$

$n^4 = \dfrac{73e}{180} 10^6$

$\boxed{n = 34}$