MTH 174 Homework #6 Key

1a. $r(t) = \sqrt{4-t^2}\,\hat{i} + t^2\hat{j} - 6t\hat{k}$ Domains

$4-t^2 \geq 0 \quad 4 \geq t^2 \rightarrow \boxed{-2 \leq t \leq 2}$

b. $\vec{r}(t) = 3\cos t\,\hat{i} + 2\sin t\,\hat{j} + t^2\hat{k}$

$\boxed{(-\infty, \infty)}$

c. $\vec{r}(t) = (1-t)\hat{i} + \sqrt{t}\,\hat{k}$ $\boxed{t \geq 0}$

d. $\vec{r}(t) = (\ln t - 1)\hat{i} + t\hat{j}$ $\boxed{t > 0}$

e. $\vec{r}(t) = \sqrt[3]{t}\,\hat{i} + \frac{1}{t+1}\hat{j} + (t+2)\hat{k}$ $\boxed{t \neq -1}$

a. $\|\vec{r}(t)\| = \sqrt{4-t^2+t^4+36t^2} = \sqrt{t^4+35t^2+4}$

b. $\|\vec{r}(t)\| = \sqrt{(3\cos t)^2+(2\sin t)^2+t^4} = \sqrt{9\cos^2 t+4\sin^2 t+t^4}$

c. $\|\vec{r}(t)\| = \sqrt{(1-t)^2+(\sqrt{t})^2} = \sqrt{t^2-2t+1+t} = \sqrt{t^2-3t+1}$

d. $\|\vec{r}(t)\| = \sqrt{(\ln t-1)^2+t^2} = \sqrt{\ln^2 t - 2\ln t + 1 + t^2}$

e. $\|r'(t)\| = \sqrt{t^{2/3}+\frac{1}{(t+1)^2}+t^2+4t+4}$

See attached for graphs

2. a. $x' = 2t \quad y' = 2 \quad s = \int_0^2 \sqrt{4t^2+4}\,dt \approx 5.92$

b. $x' = -3\cos^2\theta\sin\theta,\ y' = 3\sin^2\theta\cos\theta$

$s = \int_0^{2\pi} a\sqrt{9\cos^4\theta\sin^2\theta + 9\sin^4\theta\cos^2\theta}\,d\theta = 3a\int_0^{2\pi}|\cos\theta\sin\theta|\,d\theta$

$\sqrt{9\cos^2\theta\sin^2\theta(\cancel{\cos^2\theta+\sin^2\theta})}$ $= \cancel{0}$ $= 2(3a) = 6a$

w/o absolute value

c. $x' = e^t - e^{-t} \quad y' = -2$

$s = \int_0^3 \sqrt{e^{2t}-2+e^{-2t}+4}\,dt = \int_0^3 \sqrt{e^{2t}+2+e^{-2t}}\,dt = \int_0^3 \sqrt{(e^t+e^{-t})^2}\,dt$

$= \int_0^3 e^t+e^{-t}\,dt \approx 20.04$

d. $x' = 3,\ y' = 3\sinh 3t \quad [0,1]$

$s = \int_0^1 \sqrt{9+9\sinh^2 3t}\,dt = \int_0^1 3\sqrt{1+\sinh^2 3t}\,dt = \int_0^1 3\cosh 3t\,dt \approx 10.02$

2

2e. $x' = \frac{1}{\sqrt{1-t^2}}$ $y' = \frac{-\cancel{2}t}{1-t^2} \cdot \frac{1}{\cancel{2}}$ $s = \int_0^{1/2} \sqrt{\frac{1}{1-t^2} + \frac{t^2}{(1-t^2)^2}}\, dt \approx .55$

f. $x' = -3\sin t + 3\sin 3t$ $y' = 3\cos t - 3\cos 3t$

$s = \int_0^{\pi} \sqrt{(-3\sin t + 3\sin 3t)^2 + (3\cos t - 3\cos 3t)^2} =$

$\int_0^{\pi} \sqrt{9\sin^2 t - 18\sin t\sin 3t + 9\sin^2 3t + 9\cos^2 t - 18\cos t\cos 3t + 9\cos^2 3t}\, dt$

$= \int_0^{\pi} \sqrt{18 - 18\sin t\sin 3t - 18\cos t\cos 3t}\, dt = 12$

g. $x' = -\sin t + \frac{1}{\tan\frac{1}{2}t} \cdot \sec^2\frac{1}{2}t \cdot \frac{1}{2}$ $y = \cos t$

$\frac{1}{2}\frac{\cancel{\cos\frac{1}{2}t}}{\sin\frac{1}{2}t} \cdot \frac{1}{\cos^{\cancel{2}}\frac{1}{2}t} = \frac{1}{2}\csc\frac{1}{2}t\sec\frac{1}{2}t = \frac{1}{2\sin\frac{1}{2}t\cos\frac{1}{2}t} = \frac{1}{\sin t} = \csc t$

$s = \int_{\pi/4}^{3\pi/4} \sqrt{(-\sin t + \frac{1}{2}\csc\frac{1}{2}t\sec\frac{1}{2}t)^2 + \cos^2 t}\, dt = \int_{\pi/4}^{3\pi/4} \sqrt{(\csc t - \sin t)^2 + \cos^2 t}\, dt$

$= \int_{\pi/4}^{3\pi/4} \sqrt{\csc^2 t - 2 + \underbrace{\sin^2 t + \cos^2 t}_{=1}}\, dt = \int_{\pi/4}^{3\pi/4} \sqrt{\csc^2 t - 1}\, dt = \int_{\pi/4}^{3\pi/4} \sqrt{\cot^2 t}\, dt$

$= \int_{\pi/4}^{3\pi/4} |\cot t|\, dt = \ln|\sin t| \Big|_{\pi/4}^{3\pi/4} \approx .69$

3. $y' = -.68996 \sinh(0.0100333x)$

$s = \int_{-299.2239}^{299.2239} \sqrt{(-.68996)^2 \sinh^2(.0100333x) + 1}\, dt \approx 1480.28$

4. a. $r = 2\cos\theta$ $[0, \pi]$ $s = \int_a^b \sqrt{r^2 + (r')^2}\, d\theta$

$r' = -2\sin\theta$

$s = \int_0^{\pi} \sqrt{4\cos^2\theta + 4\sin^2\theta}\, d\theta = \int_0^{\pi} 2\, d\theta = 2\pi$

b. $r = \theta^2$ $[0, 2\pi]$ $s = \int_0^{2\pi} \sqrt{\theta^4 + 4\theta^2}\, d\theta \approx 92.90$

$r' = 2\theta$

c. $r = 2(1 + \cos\theta)$ $s = \int_0^{2\pi} \sqrt{4 + 8\cos\theta + 4\cos^2\theta + 4\sin^2\theta}\, d\theta =$

$r' = -2\sin\theta$

$\int_0^{2\pi} \sqrt{8 + 8\cos\theta}\, d\theta \approx 16$

4d. $r = \sin(6\sin\theta)$

$r' = \cos(6\sin\theta)\cdot 6\cos\theta$

$S = \int_0^{\pi/2} \sqrt{\sin^2(6\sin\theta) + \cos^2(6\sin\theta)\cdot 36\cos^2\theta}\, d\theta \approx 4.00$

e. $r = 5^\theta \quad [0, 2\pi]$

$r' = 5^\theta \ln 5$

$S = \int_0^{2\pi} \sqrt{5^{2\theta} + 5^{2\theta}(\ln 5)^2} = \int_0^{2\pi} 5^\theta \sqrt{1 + (\ln 5)^2}\, d\theta$

$\approx 29{,}015.56$

f. $r = \frac{1}{\theta}$

$r' = \frac{-1}{\theta^2}$

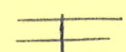
$S = \int_\pi^{2\pi} \sqrt{\frac{1}{\theta^2} + \frac{1}{\theta^4}}\, d\theta \approx .71$

g. $r = \tan\theta$

$r' = \sec^2\theta$

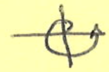
$S = \int_0^{\pi/4} \sqrt{\tan^2\theta + \sec^4\theta}\, d\theta \approx 1.07$

5a. $y = 4 \rightarrow r\sin\theta = 4 \rightarrow r = 4\csc\theta$



b. $y^2 = 9x \rightarrow r^{\cancel{2}}\sin^2\theta = 9\cancel{r}\cos\theta \rightarrow r = \frac{9\cos\theta}{\sin^2\theta} = 9\cot\theta\csc\theta$

c. $r = \theta \rightarrow \sqrt{x^2+y^2} = \tan^{-1}(y/x)$



d. $r = \frac{2}{1+\cos\theta} \rightarrow r + r\cos\theta = 2 \rightarrow \sqrt{x^2+y^2} + x = 2 \rightarrow$

$\sqrt{x^2+y^2} = 2 - x$

$\cancel{x^2} + y^2 = (2-x)^2 = 2 - 4x + \cancel{x^2}$

$y^2 = 2 - 4x$

e. $xy = 4$

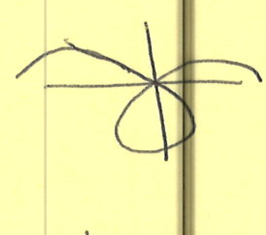
$r^2\cos\theta\sin\theta = 4 \rightarrow r^2 = 4\sec\theta\csc\theta$

f. $r = 3 \rightarrow r^2 = 9 \rightarrow x^2 + y^2 = 9$

g. $r = 5(1 - 2\sin\theta)$

$\cancel{x^2} = \quad r^2 = 5r - 10r\sin\theta$

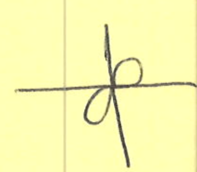
$x^2 + y^2 = 5\sqrt{x^2+y^2} - 10y$



h. $r^2 = \sin 2\theta = 2\sin\theta\cos\theta$

$r^4 = 2r\sin\theta \cdot r\cos\theta$

$(x^2+y^2)^2 = 2xy$



6a.

$$A=\frac{1}{2}\int_{-\pi/6}^{\pi/6} 4\cos^2 3\theta\,d\theta = \int_{-\pi/6}^{\pi/6} 1+\cos 6\theta\,d\theta = \theta + \frac{1}{6}\cancel{\sin 6\theta}^{\,0}\Big|_{-\pi/6}^{\pi/6} = \boxed{\frac{\pi}{3}}$$

4

b.

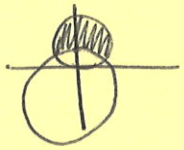
$$4-6\sin\theta=0$$
$$4=6\sin\theta$$
$$\theta=\sin^{-1}\left(\tfrac{4}{6}\right)=\sin^{-1}\left(\tfrac{2}{3}\right) \text{ and } \pi-\sin^{-1}\left(\tfrac{2}{3}\right)$$

$$A=\frac{1}{2}\int_{\sin^{-1}(\frac{2}{3})}^{\pi-\sin^{-1}(\frac{2}{3})} (4-6\sin\theta)^2\,d\theta = \frac{1}{2}\int_{\sin^{-1}(\frac{2}{3})}^{\pi-(\sin^{-1}\frac{2}{3})} 16-48\sin\theta+36\sin^2\theta\,d\theta$$

$$=\int_{\sin^{-1}(\frac{2}{3})}^{\pi-\sin^{-1}(\frac{2}{3})} 8-24\sin\theta+9(1-\cos 2\theta)\,d\theta = \int_{\sin^{-1}(\frac{2}{3})}^{\pi-\sin^{-1}(\frac{2}{3})} 17-24\sin\theta-9\cos 2\theta\,d\theta =$$

$$17\theta+24\cos\theta-\frac{9}{2}\sin 2\theta\Big|_{\sin^{-1}(\frac{2}{3})}^{\pi-\sin^{-1}(\frac{2}{3})} \approx 1.76$$

c.

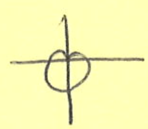


$$3\sin\theta = 2-\sin\theta$$
$$4\sin\theta=2 \qquad \theta=\pi/6,\ 5\pi/6$$
$$\sin\theta=\tfrac{1}{2}$$

$$A=\frac{1}{2}\int_{\pi/6}^{5\pi/6} (3\sin\theta)^2-(2-\sin\theta)^2\,d\theta$$

$$\approx 5.20$$

d.

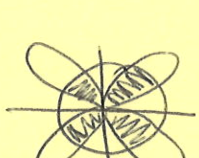


$$A=\frac{1}{2}\int_0^{2\pi}(1-\sin\theta)^2\,d\theta = \frac{1}{2}\int_0^{2\pi} 1-2\sin\theta+\sin^2\theta\,d\theta =$$

$$\frac{1}{2}\int_0^{2\pi} 1-2\sin\theta+\frac{1}{2}(1-\cos 2\theta)\,d\theta = \frac{1}{2}\int_0^{2\pi}\frac{3}{2}-2\sin\theta-\frac{1}{2}\cos 2\theta\,d\theta =$$

$$\frac{1}{2}\left[\frac{3}{2}\theta+\cancel{2\cos\theta}^{\,0}-\cancel{\frac{1}{4}\sin 2\theta}^{\,0}\right|_0^{2\pi} = \frac{3}{4}(2\pi) = \boxed{\frac{3\pi}{2}}$$

e.



$$4\sin 2\theta=2 \qquad 2\theta=\pi/6,\ 5\pi/6$$
$$\sin 2\theta=\tfrac{1}{2} \qquad \theta=\pi/12\ \ 5\pi/12$$

$$A=4*\frac{1}{2}\int_{\pi/12}^{5\pi/12}(2)^2\,d\theta = 2\int_{\pi/12}^{5\pi/12}4\,d\theta =$$

$$=2(8\theta)\Big|_{\pi/12}^{5\pi/12} = 16(\pi/3)$$

$$\frac{16\pi}{3}+1.449 \approx 18.20$$

$$4\left[\frac{1}{2}\int_0^{\pi/12}(4\sin 2\theta)^2\,d\theta+\frac{1}{2}\int_{5\pi/12}^{\pi/2}(4\sin 2\theta)^2\,d\theta\right] \approx \left(.181172+.362344/2\right)\times 4 = 1.449$$

7a. $= i$ $\quad b = iii$ $\quad c = iv$ $\quad d = v$ $\quad e = ii$

8a. $(2\cos \pi/3, 2\sin \pi/3) = (1, \sqrt{3})$

b. $(-\sqrt{2}\cos 5\pi/4, -\sqrt{2}\sin 5\pi/4) = (1, 1)$

c. $(-3\cos \pi/6, -3\sin \pi/6) = \left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

d. $(\cos 7\pi/4, \sin 7\pi/4) = \left(1/\sqrt{2}, -\frac{1}{\sqrt{2}}\right)$

e. $\left(2\cos\left(\frac{-2\pi}{3}\right), 2\sin(-2\pi/3)\right) = (-1, -\sqrt{3})$

9. a. $r = \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{-2}{2} \Rightarrow \theta = -\pi/4$ $\qquad (2\sqrt{2}, -\pi/4)$ or $(2\sqrt{2}, 7\pi/4)$

b. $(1, -2)$ $\quad r = \sqrt{1+4} = \sqrt{5}$

$\tan\theta = \left(\frac{-2}{1}\right) \Rightarrow \theta \approx -1.107$ radians $\qquad (\sqrt{5}, -1.107)$ or $(\sqrt{5}, 5.176)$

c. $(-1, \sqrt{3})$ $\quad r = \sqrt{1+3} = \sqrt{4} = 2$

$\tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) \Rightarrow \theta = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$ $\qquad \left(2, \frac{2\pi}{3}\right)$ or $(-2, -\pi/3)$

10. a. $1 + \sin\theta = 3\sin\theta$

$\frac{1}{2} = \sin\theta$ $\qquad \theta = \pi/6, 5\pi/6$

b. $r^2 = \sin 2\theta = \cos 2\theta$

$2\theta = \pi/4, 5\pi/4, 9\pi/4, 13\pi/4$

$\theta = \pi/8, 5\pi/8, 9\pi/8, 13\pi/8$

c. $\cancel{1} - \cos\theta = \cancel{1} + \sin\theta$

$-1 = \tan\theta$ $\qquad \theta = \cancel{-\pi/4}, 3\pi/4, 7\pi/4$

d. $2\sin 2\theta = 1$ $\qquad 2\theta = \pi/6, 5\pi/6, 13\pi/6, 17\pi/6$

$\sin 2\theta = \frac{1}{2}$ $\qquad \theta = \pi/12, 5\pi/12, 13\pi/12, 17\pi/12$

e. $\cos 3\theta = \sin 3\theta$

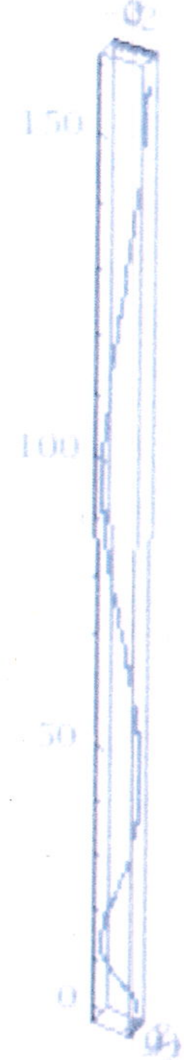
$3\theta = \pi/4, 5\pi/4, 9\pi/4, 13\pi/4, 17\pi/4, 21\pi/4$

$\theta = \pi/12, 5\pi/12, 9\pi/12, 13\pi/12, 17\pi/12, 21\pi/12$

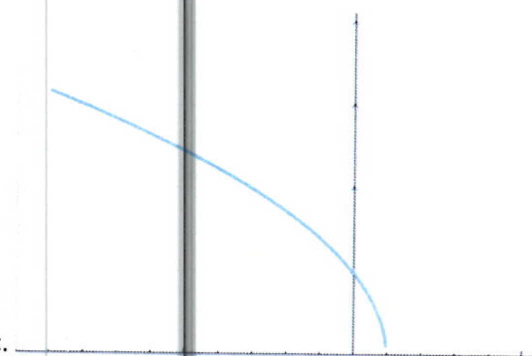
$9\pi/12 = 3\pi/4$ $\qquad 21\pi/12 = 7\pi/4$

11a. ellipse – see attached for graphs
b. ellipse
c. ellipse
d. hyperbola
e. ellipsoid (ellipse in xy + z^2 for 3D)
f. ellipsoid (ellipse in yz + x^2 for 3D)

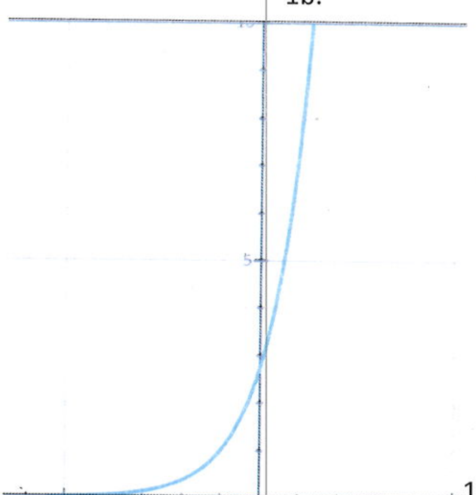
1a.



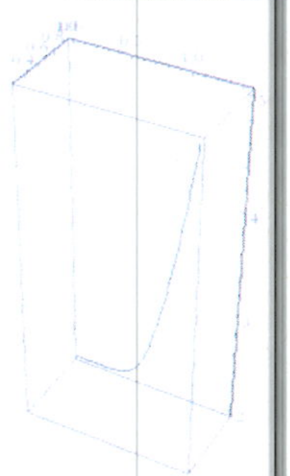
1b.



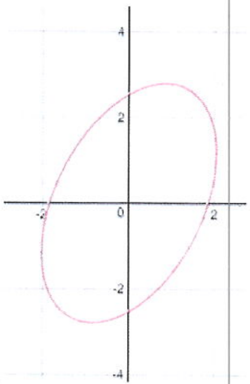
1c.



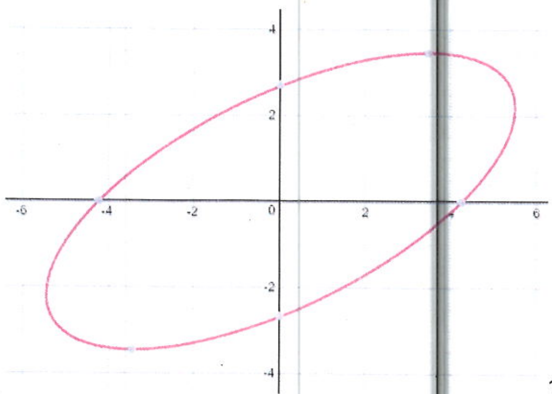
1d.



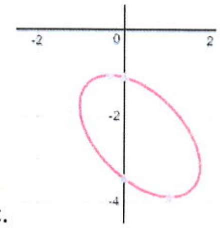
1e.



11a.

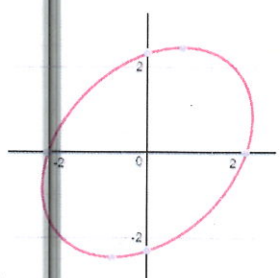


11b.

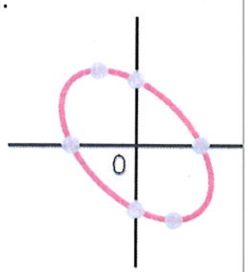


11c.

11d.



11e. (xy-plane)



11f. (yz-plane)