

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Determine the interval(s) for  $x$  for which the series converges. State the radius of convergence. Be sure to check the endpoints.

a.  $\sum_{n=0}^{\infty} \left(\frac{x-3}{5}\right)^n$

$$\left| \frac{x-3}{5} \right| < 1$$

$$-1 < \frac{x-3}{5} < 1$$

$$\frac{-5}{+3} < x-3 < \frac{5}{+3}$$

b.  $\sum_{n=0}^{\infty} \frac{(x+1)^n}{n!}$

$$\lim_{n \rightarrow \infty} \left| \frac{(x+1)^{n+1}}{(n+1)!} \right|$$

$$\frac{n!}{(x+1)^n} = \lim_{n \rightarrow \infty} \frac{x+1}{n+1} = 0$$

$$R = \infty$$

$$\text{Interval } (-\infty, \infty)$$

Radius of convergence = 5

Interval of convergence  $(-2, 8)$

@ -2  $\left(\frac{-5}{5}\right)^n = (-1)^n$  diverges

@ 8  $\left(\frac{5}{5}\right)^n = 1^n$  diverges.

2. Determine the convergence or divergence of the series. State the test that was used.

a.  $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

Converges

use direct comparison or telescoping  
or limit comparison  
on  $\frac{1}{n^2}$

b.  $\sum_{n=0}^{\infty} \left(\frac{\pi}{e}\right)^n$

diverges geometric  $\frac{\pi}{e} > 1$

c.  $\sum_{n=1}^{\infty} \frac{\sqrt{n-5}}{n^2+1}$

Converges direct comparison or limit comparison  
w/  $\frac{1}{n^{3/2}}$

d.  $\sum_{n=1}^{\infty} \frac{2+\sin n}{n}$

diverges limit comparison of squeeze  
theorem

e.  $\sum_{n=0}^{\infty} \frac{\cos n\pi}{n+1}$

=  $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}$

converges alternating series