MTH 277, Exam #1, Spring 2018 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing" for the answer only; partial credit will not be possible and any credit awarded for the work will not be available.

1. Use $\vec{u} = \langle 4,2,-7\rangle$, $\vec{v} = \langle 2,-3,-1\rangle$ to find the following. (4 points each)
   a. $\vec{u} + \vec{v}$

   $\langle 6, -1, -8\rangle$

   b. $\|\vec{u}\|$

   $\sqrt{16+4+49} = \sqrt{69}$

   c. Write a unit vector in the direction of $\vec{u}$

   $\left\langle \frac{4}{\sqrt{69}}, \frac{2}{\sqrt{69}}, -\frac{7}{\sqrt{69}} \right\rangle$

   d. Find $\vec{u} \cdot \vec{v}$

   $8-6+7 = 9$

   e. Find the angle between $\vec{u}$ and $\vec{v}$

   $\cos^{-1}\left(\frac{9}{\sqrt{69}\cdot\sqrt{14}}\right) = 1.28$ radians

   $73.2°$

   f. Find $\vec{u} \times \vec{v}$

   $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & -7 \\ 2 & -3 & -1 \end{vmatrix} = (-2-21)\hat{i} - (-4+14)\hat{j} + (-12-4)\hat{k}$

   $= \langle -23, -10, -16\rangle$

2. Find the volume of the parallelepiped bounded by the vectors $\vec{u} = \langle 1,3,1\rangle, \vec{v} = \langle 0,6,6\rangle,\ \vec{w} = \langle -4,0,-4\rangle$. (5 points)

$$\begin{vmatrix} 1 & 3 & 1 \\ 0 & 6 & 6 \\ -4 & 0 & -4 \end{vmatrix} = (-24-0)(1) - (0+24)(3) + (0+24)(1)$$

$$= -\cancel{24} - 72 + \cancel{24} = -72$$

$$\text{Volume} = |-72| = \boxed{72}$$

3. State the domain and range of the function $f(x, y) = \sqrt{4 - x^2 - 4y^2}$ in appropriate notation. (6 points)

$$D: \{(x,y) \mid x^2 + 4y^2 \le 4\}$$

$$R: [0, 2]$$

4. Find the equation of the plane perpendicular to the line $\frac{x-1}{-2} = y - 4 = z$, passing through the point $(2, -3, 1)$. (6 points)

$$\langle -2, 1, 1\rangle$$

$$-2(x-2) + 1(y+3) + 1(z-1) = 0$$

5. Identify the quadric surface $z^2 - x^2 - \frac{y^2}{4} = 1$, and convert the equation to cylindrical and spherical coordinates. (10 points)

Hyperboloid of 2 sheets

$$\rho^2\cos^2\varphi - \rho^2\sin^2\varphi\cos^2\theta - \frac{\rho^2\sin^2\varphi\sin^2\theta}{4} = 1$$

$$\rho^2 = \frac{1}{\cos^2\varphi - \sin^2\varphi(\cos^2\theta + 1/4\sin^2\theta)} \quad \text{Spherical}$$

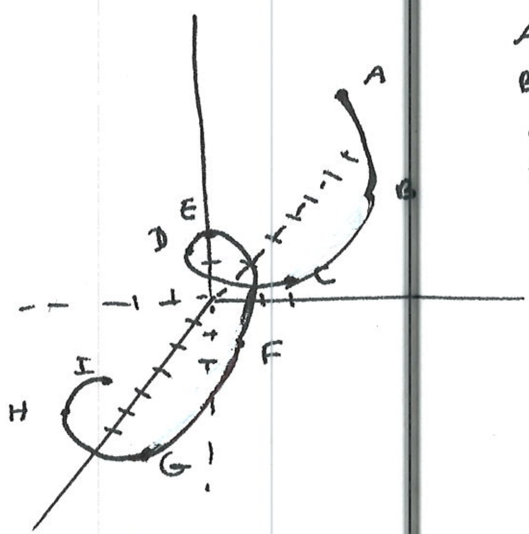
$$z^2 - r^2\cos^2\theta - \frac{1}{4}r^2\sin^2\theta = 1$$

$$z^2 = 1 + r^2(\cos^2\theta + \frac{1}{4}\sin^2\theta) \quad \text{cylindrical}$$

6. Write an equation of the cylinder $4x^2 + y^2 = 16$ in parametric (surface) form. (6 points)

$$\vec{r}(u,v) = 2\cos u\,\hat{i} + 4\sin u\,\hat{j} + v\,\hat{k} \qquad \frac{x^2}{4} + \frac{y^2}{16} = 1$$

7. Sketch the graph of $\vec{r}(t) = t\hat{\imath} + 2\sin t\,\hat{\jmath} + 2\cos t\,\hat{k}$ for two cycles. (10 points)



	t	x	y	z
A	-2π	-6.3	0	2
B	$-3\pi/2$	-4.7	2	0
C	$-\pi$	-3.1	0	-2
D	$-\pi/2$	-1.6	-2	0
E	0	0	0	2
F	$\pi/2$	1.6	2	0
G	π	3.1	0	-2
H	$3\pi/2$	4.7	-2	0
I	2π	6.3	0	2

8. Consider $\vec{u}(t) = \sec^2 t\,\hat{\imath} + \frac{1}{1+t^2}\hat{\jmath} + t^{3/2}\hat{k}$. Find: (5 points each)

a. $\vec{u}'(t)$

$$2\sec t \cdot \sec t \tan t\,\hat{\imath} + \frac{-2t}{(1+t^2)^2}\hat{\jmath} + \frac{3}{2}t^{1/2}\hat{k}$$

$$= \boxed{2\sec^2 t \tan t\,\hat{\imath} - \frac{2t}{(1+t^2)^2}\hat{\jmath} + \frac{3}{2}\sqrt{t}\,\hat{k}}$$

b. $\int \vec{u}(t)\,dt$

$$(\tan t + C_1)\hat{\imath} + (\arctan t + C_2)\hat{\jmath} + \left(\frac{2}{5}t^{5/2} + C_3\right)\hat{k}$$

c. Describe the continuity of $\vec{u}(t)$.

X term undefined at multiples (odd) of $\pi/2$

Z term $t \geq 0$

9. Find the limits. (6 points each)

a. $\lim_{(x,y)\to(0,0)} \frac{x^2+2xy^2+y^2}{x^2+y^2}$

let
$y = kx$

$$\lim_{x\to 0} \frac{x^2+2x\,k^2x^2+k^2x^2}{x^2+k^2x^2} = \lim_{x\to 0} \frac{\cancel{x^2}(1+2k^2x+k^2)}{\cancel{x^2}(1+k^2)}$$

$$= \lim_{x\to 0} \frac{1+2k^2x+k^2}{1+k^2} = \frac{1+k^2}{1+k^2} = \boxed{1}$$

b. $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^3+y^2}$

$x^3 = y^2$

$x^{3/2} = y$

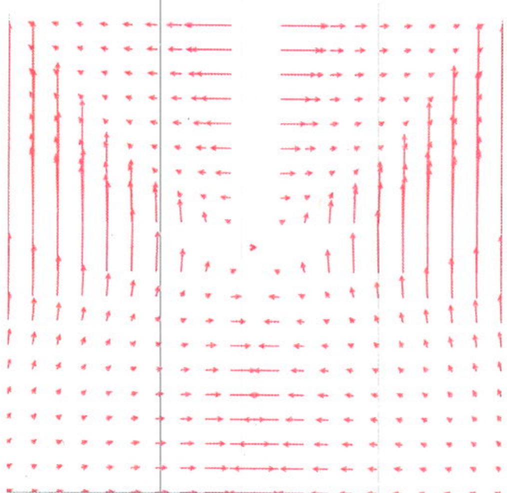
let $y = kx^{3/2}$

$$\lim_{x\to 0} \frac{x^2\cdot kx^{3/2}}{x^3+(kx^{3/2})^2} = \lim_{x\to 0} \frac{x^{7/2}}{x^3+k^2x^3} = \lim_{x\to 0} \frac{\cancel{x^3}\cdot x^{1/2}}{\cancel{x^3}(1+k^2)} = \boxed{0}$$

10. Identify which vector field goes with each graph. (4 points each)

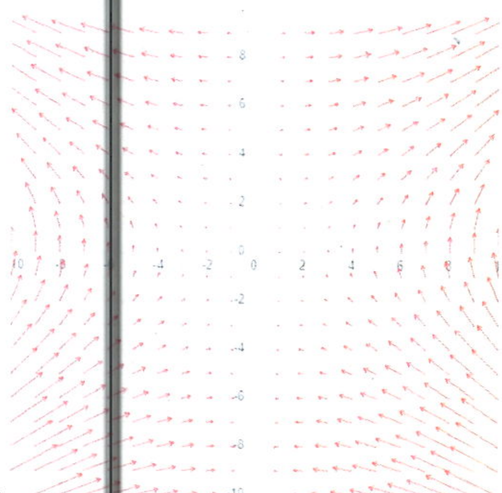
a. $\vec{F}(x,y) = 2xy\hat{\imath} + x^2\hat{\jmath}$ ii

b. $\vec{F}(x,y) = \frac{2y}{x}\hat{\imath} + \frac{x^2}{y^2}\hat{\jmath}$ i



i.

B



ii.

A

11. Find the value of the line integral $\int_C (x+4\sqrt{y})ds$ on the path between (2,3) and (3,4). (10 points)

$\langle 1,1 \rangle$

$\vec{r}(t) = (t+2)\hat{i} + (t+3)\hat{j}$

$t \in [0,1]$

$\vec{r}'(t) = 1\hat{i} + 1\hat{j}$

$ds = \|\vec{r}'\| = \sqrt{1+1} = \sqrt{2}\,dt$

$$\int_0^1 (t+2+4\sqrt{t+3})\sqrt{2}\,dt$$

$$= \frac{1}{2}t^2 + 2t + 4\cdot\frac{2}{3}(t+3)^{3/2}\Big|_0^1$$

$$= \frac{1}{2}(1) + 2(1) + \frac{8}{3}(4)^{3/2} - 0 - 0 - \frac{8}{3}\sqrt{3}$$

$$\frac{1}{2} + 2 + \frac{64}{3} - \frac{8\sqrt{3}}{3} = \boxed{\frac{143}{6} - \frac{8\sqrt{3}}{3}}$$

12. Find all first partial derivatives of $f(x,y,z) = z^2y^3 + z\sin(x+y) + \sqrt{y^2 - z^2}$. (9 points)

$f_x = z\cos(x+y)$

$f_y = 3y^2z^2 + z\cos(x+y) + \frac{1}{2}(y^2-z^2)^{-1/2}\cdot 2y$

$f_z = 2y^3z + \sin(x+y) + \frac{1}{2}(y^2-z^2)^{-1/2}(-2z)$

13. Find the total differential of $w = x^2yz^2 + \sin(yz)$. (7 points)

$f_x = 2xyz^2$

$f_y = x^2z^2 + \cos(yz)\cdot z$

$f_z = 2x^2yz + \cos(yz)\cdot y$

$\Delta w = 2xyz^2\cdot\Delta x + (x^2z^2 + z\cos yz)\Delta y + (2x^2yz + y\cos yz)\Delta z$