TH 277 Homework #1 Key



1. a. $\|\vec{v}\| = 5,\ \theta = 120°$ $\quad \cos\theta = -\frac{1}{2} \quad \sin\theta = \frac{\sqrt{3}}{2}$

$\vec{v} = 5(-\frac{1}{2})\hat{i} + 5(\frac{\sqrt{3}}{2})\hat{j} = -\frac{5}{2}\hat{i} + \frac{5\sqrt{3}}{2}\hat{j} = \boxed{\langle -5/2, \frac{5\sqrt{3}}{2} \rangle}$

b. $\|\vec{v}\| = 8,\ \theta = -3.5$ rad $\quad \cos\theta = -.9364566873$

$\sin\theta = .3507832277$

$\vec{v} = 8(-.9364566873)\hat{i} + 8(.3507832277)\hat{j} = \boxed{-7.49\hat{i} + 2.806\hat{j}}$

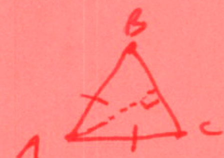
$\langle -7.49, 2.806 \rangle$

2. $\|\vec{v}\| = \sqrt{20^2 + 20^2} = \sqrt{800} = \boxed{20\sqrt{2}}$

$\theta = \pi/4 = \boxed{45°}$

3. $A(5,3,4), B(7,1,3), C(3,5,3)$

i. $\vec{AB} = \langle 2,-2,-1 \rangle$
$\vec{BA} = \langle -2,2,1 \rangle$
$\vec{BC} = \langle -4,4,0 \rangle$
$\vec{CB} = \langle 4,-4,0 \rangle$
$\vec{AC} = \langle -2,2,-1 \rangle$
$\vec{CA} = \langle 2,-2,1 \rangle$




ii. $\|\vec{AB}\| = \sqrt{4+4+1} = \sqrt{9} = 3$
$\|\vec{BC}\| = \sqrt{16+16+0} = 4\sqrt{2}$
$\|\vec{AC}\| = \sqrt{4+4+1} = \sqrt{9} = 3$

iii. $\boxed{\text{isosceles}}$ since $\vec{AB}$ and $\vec{AC}$ are the same length.

iv. $\left(\frac{7+3}{2}, \frac{1+5}{2}, \frac{3+3}{2}\right) = \left(\frac{10}{2}, \frac{6}{2}, \frac{6}{2}\right) = \boxed{(5,3,3)} = m$

v. $\vec{AB} \cdot \vec{AC} = -4-4+1 = -7$ $\quad \cos^{-1}\left(\frac{-7}{3\cdot 3}\right) = 2.462$ radians
$141.1°$

$\boxed{\text{Obtuse.}}$

vi. $\vec{h} = \langle 0,0,1 \rangle$ $\quad \|h\| = 1$ $\quad A = \frac{1}{2}bh = \frac{1}{2}\cdot(1)(4\sqrt{2}) = 2\sqrt{2}$
$= \vec{Am}$

or $\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 2 & -1 \\ 2 & -2 & -1 \end{vmatrix} = (-2-2)\hat{i} - (2+2)\hat{j} + (4-4)\hat{k}$
$-4\hat{i} - 4\hat{j} + 0\hat{k}$

$\frac{1}{2}\|\vec{AB} \times \vec{AC}\| = \frac{1}{2}\sqrt{16+16+0} = \frac{1}{2}\cdot 4\sqrt{2} = \boxed{2\sqrt{2}}$

4a. $(x^2+6x+9) + (y^2-2y+1) + (z^2+10z+25) = 19+9+1+25$
$(x+3)^2 + (y-1)^2 + (z+5)^2 = 54$

$\boxed{\text{Center } (-3,1,-5) \quad \text{radius} = \sqrt{54}}$

46. $2x^2+2y^2+2z^2 = 8x-24z+1$

$2(x^2-4x+4)+2(y^2)+2(z^2+12z+36) = 1+72+8$

$2(x-2)^2+2y^2+2(z+6)^2 = 81$

$(x-2)^2+y^2+(z+6)^2 = \frac{81}{2}$

$\boxed{\text{Center } (2,0,-6) \quad \text{radius} = \frac{9}{\sqrt{2}}}$

c. center $(2,-4,1)$ radius $r=5$

$\boxed{(x-2)^2+(y+4)^2+(z-1)^2 = 25}$

d. $(2,0,0)$, $(0,6,0)$ diameter

$\left(\frac{2+0}{2}, \frac{0+6}{2}, \frac{0+0}{2}\right) = (1,3,0)$ center (midpoint of diameter)

radius = distance from center to point on circle

$\|\langle -1,3,0\rangle\| = \sqrt{1+9} = \sqrt{10}$

$\boxed{(x-1)^2+(y-3)^2+z^2 = 10}$

5. $\|\vec{u}\| = \sqrt{16+9+25} = \sqrt{50} = 5\sqrt{2}$

$\alpha = \cos^{-1}\left(\frac{-4}{5\sqrt{2}}\right) = \boxed{124.4°}$ 2.172 radians

$\beta = \cos^{-1}\left(\frac{3}{5\sqrt{2}}\right) = \boxed{64.9°}$ 1.133 radians

$\gamma = \cos^{-1}\left(\frac{5}{5\sqrt{2}}\right) = \boxed{45°}$ $\pi/4$

6. $\|F_1\| = 30N$, $\angle F_1 = -120°$ $\quad F_1 = \langle 30(-1/2), 30(-\frac{\sqrt{3}}{2})\rangle = \langle -15, -15\sqrt{3}\rangle$

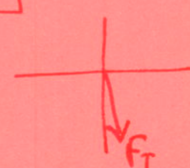
$\|F_2\| = 5N$, $\angle F_2 = -60°$ $\quad F_2 = \langle 5(1/2), 5(-\frac{\sqrt{3}}{2})\rangle = \langle 5/2, -\frac{5\sqrt{3}}{2}\rangle$

$\|F_3\| = 20N$, $\angle F_3 = 45°$ $\quad F_3 = \langle 20(\frac{\sqrt{2}}{2}), 20(\frac{\sqrt{2}}{2})\rangle = \langle 10\sqrt{2}, 10\sqrt{2}\rangle$

$F_1+F_2+F_3 = \langle -15+5/2+10\sqrt{2}, -15\sqrt{3} + -\frac{5\sqrt{3}}{2} + 10\sqrt{2}\rangle \approx \langle 1.642, -16.169\rangle = F_T$

$\|F_1+F_2+F_3\| = \|F_T\| = \sqrt{1.642^2+(-16.169)^2} = \sqrt{264.12} \approx \boxed{16.25}$

$\theta = \tan^{-1}\left(\frac{-16.169}{1.642}\right) \approx \boxed{-84.2°}$, $-1.47$ radians




7. a. $\vec{u}\cdot\vec{v} = 5-6-12 = -13$

$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 4 \\ 5 & -2 & -3 \end{vmatrix} = (-9+8)\hat{i} - (-3-20)\hat{j} + (-2-15)\hat{k}$

$\boxed{-1\hat{i} + 23\hat{j} - 17\hat{k}}$

7b. $\vec{u}\cdot\vec{v} = 27 + 8 - 5 = 30$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 9 & -4 & 1 \\ 3 & -2 & -5 \end{vmatrix} = (20+2)\hat{i} - (-45-3)\hat{j} + (-18+12)\hat{k}$$

$$\boxed{22\hat{i} + 48\hat{j} - 6\hat{k}}$$

8. $$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8 & -6 & 4 \\ 10 & -12 & -2 \end{vmatrix} = (12+48)\hat{i} - (16-40)\hat{j} + (96+60)\hat{k}$$

$$60\hat{i} + 24\hat{j} + 156\hat{k}$$

$$\div 12 \quad 5\hat{i} + 2\hat{j} + 13\hat{k}$$

$$\| \langle 5, 2, 13 \rangle \| = \sqrt{25+4+169} = \sqrt{198} = 3\sqrt{22}$$

$$\hat{u} = \boxed{\left\langle \frac{5}{\sqrt{198}}, \frac{2}{\sqrt{198}}, \frac{13}{\sqrt{198}} \right\rangle}$$

9. $\vec{u} = \langle 2-1, 3-1, 4-1 \rangle = \langle 1, 2, 3 \rangle$

$\vec{v} = \langle 6-1, 5-1, 2-1 \rangle = \langle 5, 4, 1 \rangle$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 4 & 1 \\ 6 & 6 & 4 \end{vmatrix} = \hat{i}(\underbrace{16-6}_{10}) - \hat{j}(\underbrace{20-6}_{14}) + \hat{k}(\underbrace{30-24}_{6}) =$$

$$10\hat{i} - 14\hat{j} + 6\hat{k}$$

$$\| \vec{u}\times\vec{v} \| = \sqrt{100+196+36} = \sqrt{332} = \boxed{2\sqrt{83}}$$

10. $$\vec{u}\cdot(\vec{v}\times\vec{w}) = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = 2(4+1) = 2(5) = \boxed{10}$$

11. $\vec{u}\cdot\vec{v} = 3 - 4 - 6 = -7$ $\qquad \|\vec{v}\| = \sqrt{9+16+4} = \sqrt{29}$

$$\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \left(\frac{-7}{29}\right)\langle 3, 4, -2 \rangle$$

$$= \boxed{\left\langle \frac{-21}{29}, \frac{-28}{29}, \frac{14}{29} \right\rangle}$$

12. $\langle a, b, c \rangle \cdot \langle 4, 0, 2 \rangle = 4a + 0b + 2c = 0$

$$4a = -2c$$

$$2a = -c$$

$a = 1, \; c = -2$

$\boxed{\langle 1, 0, -2 \rangle}$ $\qquad \langle 1, 0, -2 \rangle \cdot \langle 4, 0, 2 \rangle = 4 + 0 - 4 = 0$ ✓

13. $\vec{F} \cdot \vec{d} = W$

$\vec{d} = \langle 6-0, 12-10, 20-8 \rangle = \langle 6, 2, 12 \rangle$

$\vec{F} \cdot \vec{d} = \langle 8, -6, 9 \rangle \cdot \langle 6, 2, 12 \rangle = 48 - 12 + 108 = \boxed{144 \text{ N}\cdot\text{m}}$