

**Instructions:** Show all work. Use exact answers unless otherwise asked to round.

1. Find the directional derivative on the surface  $f(x, y) = \sin 2x \cos y$  at the point  $(\pi, 0)$  in the direction of  $\langle 1, -1 \rangle$ .

$$\begin{aligned}\nabla f &= \langle 2\cos 2x \cos y, -\sin 2x \sin y \rangle \\ &= \langle 2\cos 2\pi \cdot \cos 0, -\sin(2\pi) \sin 0 \rangle \\ &= \langle 2 \cdot 1 \cdot 1, 0 \rangle = \langle 2, 0 \rangle\end{aligned}$$

$$\vec{v} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$$

$$\nabla f \cdot \vec{v} = \frac{2}{\sqrt{2}} + 0/\sqrt{2} = \boxed{\frac{2}{\sqrt{2}}}$$

$$\begin{aligned}\vec{u} &= \langle 1, -1 \rangle \\ \|\vec{u}\| &= \sqrt{1^2 + (-1)^2} = \sqrt{2} \\ \vec{v} &= \frac{\vec{u}}{\|\vec{u}\|}\end{aligned}$$

2. What is the maximum value of the directional derivative of  $f(x, y, z) = xye^z$  at the point  $(4, 3, 1)$ .

$$\begin{aligned}\nabla f &= \langle ye^z, xe^z, xye^z \rangle \\ &= \langle 3e, 4e, 12e \rangle\end{aligned}$$

$$\begin{aligned}\|\nabla f\| &= \sqrt{(3e)^2 + (4e)^2 + (12e)^2} = \sqrt{9e^2 + 16e^2 + 144e^2} = \sqrt{169e^2} \\ &= 13e\end{aligned}$$